ANNALS

of the

Association of American Geographers

Volume XXVII DECEMBER, 1937 No. 4

Some Measures of Distributions*

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* The writer owes more than the usual debt of gratitude to Mr. O. M. Miller of the American Geographical Society not only for many helpful suggestions but also for his constant encouragement and interest in this study, extending over a period of two years.

If we look at a map showing the distribution of population in the United States it is obvious at once that the population is more evenly distributed in Iowa than in New York State. If we compare two graphs for the same period of years, say 1901–1930, showing how much rain fell each year at New York City and at Yuma, Arizona, it will be seen that the total rainfall was more evenly distributed among the years at New York than it was at Yuma. We know that income per capita is more evenly distributed among college professors than among the population at large.

Here are three typical examples of distributions. Distribution maps for areal distributions and frequency diagrams of the familiar kind for non-areal distributions may show that such distributions differ in evenness. What the maps and diagrams fail to disclose in definite quantitative terms is how much they differ. Furthermore, unless the differences are great it is difficult, if not impossible, to tell which of two distributions is the more even from mere inspection of tables, maps, or graphs. There are phases of geographical study in which it would be illuminating if we could compare the *degrees of evenness* of different distributions with the aid of precise figures.

It would also be of interest if we could express in quantitative terms the *degree of areal association* of different distributions—i.e., the degree to which two distributions either "fit together" or tend to occupy different areas. Comparison of distribution maps shows in a rough and ready manner the degree to which the distributions of different phenomena (such as population and altitude zones, rainfall and vegetation, coalfields and cities) correspond to one another, and geographers frequently draw conclusions from such rough comparisons. Some of the comparisons might be more exact and the conclusions better founded were they based on definite figures rather than on visual impressions.

In this paper, a preliminary essay in a subject believed worthy of further development, methods of measuring the degree of evenness of individual distributions and the degree of association of different distributions are discussed. All that can be done here is to outline the methods and theory and give a few examples. Particular attention will be devoted to areal distributions, the type of distribution with which geographers are likely to be most concerned. The analysis of areal distributions, however, raises complications not encountered when the elements of space or location or area are left out of the picture, and, in order that these may be understood, certain fundamental principles must be considered that apply to the measurement of all types of distributions.

The techniques, which are based largely on analysis of the so-called "Lorenz curves" of the statisticians, have been worked out so as to yield

"coefficients of evenness" and of "association." With "maximum evenness" or "maximum association" the value (or magnitude) of any coefficient is 1.0 (or 100 per cent). With "minimum evenness" or "minimum association" the value is 0. The coefficients are locational and non-locational. The locational coefficients disclose the degree of evenness or of association of distributions with respect either to lines (*linear distributions*) or to areas (*areal distributions*). In the non-locational coefficients factors of position and area are not taken into account.

GENERAL PRINCIPLES

DISTRIBUTORS AND DISTRIBUTEES

The word "distribution" implies a relationship of one element to another: population to territory, income to families, horses to farms, etc. The two elements may consist of altogether different phenomena, as in the examples just given; or else one element may consist of groups of the other one, as in the case of the distribution of persons among families or communities. In order to measure the evenness of any distribution both elements must be split up into units that may be counted or measured: persons, dollars, square miles, etc. Now it is clear that we may conceive of the units of either element as distributed among the units of the other. We ordinarily think of population as distributed among units of area, so many persons to the square miles here, so many there, etc., but we might equally well think of area as distributed among people (so many square miles per capita here, so many there). In our discussion we shall call the element that is held to be distributed the *distribution* and the element with reference to which the evenness of the distribution is measured the *distributary*. The units comprising the distribution will be referred to as *distributees*, or, for simplicity's sake, as "persons," and collectively as "population," it being understood, of course, that these terms do not necessarily signify human beings. The divisions into which the distributary is subdivided for the purposes of analysis will be called *distributors*. These divisions may be either equal or unequal parts of the distributary, and the units by which their sizes are measured will be called *secondary distributors*. (If, however, the distributors are equal, no account of their sizes need actually be taken in the analysis.) In statistical terms the "population" of a distributor and the number of secondary distributors of which it is composed are "variates."

When we analyze the evenness of the distribution of human beings among 25 houses, the 25 houses constitute the distributary, the people are the distributees, and each house is a distributor. (Secondary distributors, such as cubic feet, or rooms, in each house, need not be considered in this

case.) A coefficient showing the degree of evenness of such a distribution (i.e., a distribution among distributors deemed to equal) will be designated by the letter T (for "totals," each total being the number of distributees in each distributor).

If, on the other hand, we were to analyze the distribution of human beings *per room* among the 25 houses, all of the rooms in the 25 houses would constitute the distributary, all of the rooms in each house would constitute a distributor, each room would be a secondary distributor, and the people, as above, would be the distributees. The evenness coefficient in this case would be designated by the letter R, meaning a coefficient of the evenness of the distribution of the *ratios* of the distributees (persons) to the secondary distributors (rooms) among the distributors (houses).

Both T and R may be regarded as coefficients that show either the evenness of the distribution of one element (A) among subdivisions of another (B) or, what is merely another way of stating the same thing, the degree of association of the two elements (A and B) with one another. T shows the degree of association of the distributees with the distributors; R, the degree of association of the distributees with the secondary distributors.

Coefficients T and R are both *simple* coefficients, in that they represent the degree of evenness or of association of distributions when measured with respect to only one aggregation or *system* of distributors. *Compound* coefficients, employed in measuring the evenness or the association of certain areal distributions, are found by combining two or more simple coefficients or by introducing other additional factors, as will be explained later.

An *areal coefficient* indicates the degree of evenness of a distribution over a region or the degree of association of two distributions with respect to the area of a region. Here the distributors are established by dividing into smaller areas the region within which the distributions lie. When the evenness of an areal distribution is measured each smaller area itself constitutes an areal distributor whose magnitude is measured in terms of such secondary distributors as square miles, acres, etc. If the areal distributors are all of the same area, the degree of evenness may be shown by a type of coefficient T that we shall designate T_a (a meaning "area"), and the actual sizes of the distributors need not be considered. If, however, the areal distributors are of different areas—as would be the case were we to measure the evenness of the areal distribution of population among the counties of a State—a coefficient R_a is employed. In this case the ratios are those of population to area (i.e., densities of population) in each distributor (county).

In measuring the degree of areal association of two elements A and B,

let us call A the distribution and B the distributary. The degree of association is indicated by a type of coefficient R that will be designated R_{as} (*as* meaning "areal association"). R_{as} shows the evenness of the distribution of the ratios of distributees (units of A) to secondary distributors (units of B) among the distributors.

The simple areal coefficients T_a, R_a, and R_{as} give only partial indications of the degree of areal evenness or association. In order to obtain more complete values, E for evenness and As for association, compound coefficients must be employed, as we shall see.

THE DISTRIBUTION OF TOTALS (COEFFICIENTS T)

Provided we know the population (number of distributees) in each distributor,[1] the value of any coefficient T or T_a may be found by constructing a graph like that shown in Figure 1. Here the horizontal distance I–II

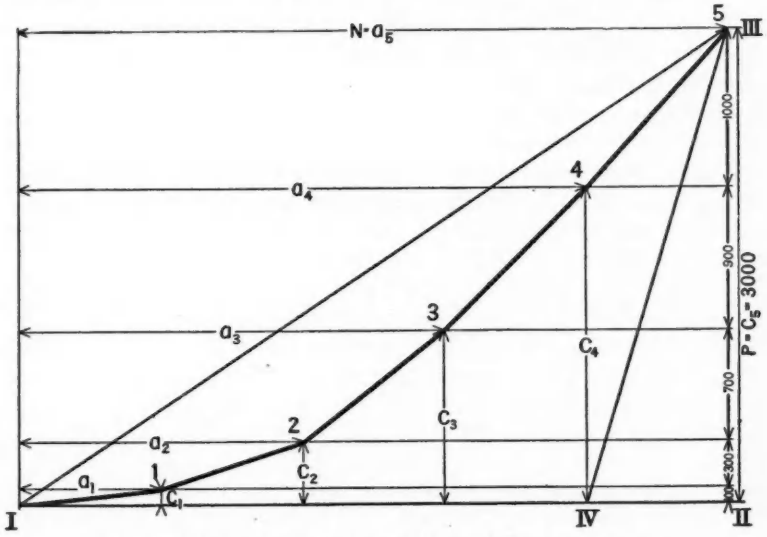

FIG. 1—Illustrating method of finding coefficients of type T.

represents the number of distributors (i.e., the size of the distributary) and the height II–III the total population (the distribution). Let us assume that we are measuring the distribution of population among five communi-

[1] In finding T_a the population of each distributor may be determined by the methods explained on pages 201–202.

ties (distributors). The communities, in increasing order of population, are represented by the five vertical columns from left to right, the widths of the columns being equal since the distributors are deemed to be equal parts of the distributary. The populations of the communities are indicated by the vertical distances between the horizontal lines in increasing order of population from bottom to top. Consequently the lengths of the vertical lines c_1, c_2, c_3, etc., represent the cumulated populations in the several communities (c_3, for example, represents the total population in the first, second, and third smallest communities, and c_5 or P, the total population). The line drawn from I to III passing through points 1, 2, 3, and 4 is a Lorenz curve,[2] or *distribution line*. If the population were evenly distributed among the communities (600 to a community) this line would coincide with the straight line I–III. Such a distribution line would indicate "maximum evenness of distribution."

What then would be the trend of a distribution line indicating "minimum evenness" or "minimum association"? The more a population tends to become concentrated at one place, the more unevenly it is distributed. The ultimate theoretical extreme of unevenness is reached only when the entire population is concentrated at one infinitely small point, or, in other words, when the population is so densely concentrated that no part of it is associated with any measurable part of any distributor. (Similarly, minimum association of two elements occurs if no part of element A (the distribution) is associated with any part of element B (the distributary).) Thus the more unevenly a population is distributed, the more closely the distribution line approaches lines I–II and II–III until, with theoretical minimum evenness, it coincides with these two lines. Minimum *theoretical* evenness is an abstract concept that can exist only in the imagination. The minimum measurable degree of evenness in the case of any actual distribution would be reached were the entire population to be concentrated in *one* distributor, and this would be represented by a distribution line coinciding with lines I–IV and IV–III.

As a basis for determining any coefficient of evenness we might take either minimum theoretical evenness or minimum measurable evenness as our starting point, with a value of 0. The former would appear to be preferable, since only in this case is the coefficient the same irrespective of which of the two elements is regarded as the distributary. If, therefore, we decide to express minimum theoretical evenness by a value of 0 and maxi-

[2] Lorenz, M. O., "Methods of Measuring the Concentration of Wealth," *Publ. of the Amer. Statistical Assn.*, New Series, Vol. 9, 1904–1905, pp. 209–219. See also Day, E. E., *Statistical Analysis*, New York, 1932, pp. 85–89.

mum evenness by unity (100 per cent), the actual degree of evenness, or coefficient T, may be expressed as the ratio to the total area of the triangle I–II–III of the area D comprised between the distribution line and lines I–II and II–III. The area D in Figure 1 is the sum of the areas of the five columns bounded by the distribution line, the base line I–II, and the lines c_1, c_2, c_3, c_4, and c_5. It will be seen that these areas are $\frac{1}{2}c_1$, $\frac{1}{2}(c_1+c_2)$, $\frac{1}{2}(c_2+c_3)$, $\frac{1}{2}(c_3+c_4)$ and $\frac{1}{2}(c_4+c_5)$, respectively, the width of each column being 1. Hence their sum is $c_1+c_2+c_3+c_4+\frac{1}{2}c_5$, or, since $c_5=P$, $\Sigma c-\frac{1}{2}P$, or $\frac{1}{2}(2\Sigma c-P)=D$. The area of triangle I–II–III is clearly $\frac{1}{2}PN$, N being the total number of distributors, represented by the length of the base I–II.

Hence the ratio of the two areas

$$T=2D/PN=(2\Sigma c-P)/PN. \qquad \text{[Formula 1]}^3$$

For any actual coefficient of evenness the number of distributors may be added as a subscript, thus T_5, T_{100}, T_{a5}, T_{a100}, etc.

In practice T may be found thus: (1) Write the population of each distributor on a separate card. (2) Arrange the cards in order of population from least to greatest. (3) Cumulate the populations, preferably with the aid of an adding machine, and tabulate the cumulated quantities for the several distributors. (4) Find the sum of the cumulated populations: Σc. (5) Work out the value of T from Formula 1.

T_{25}, the coefficient of the evenness of the distribution of population among the 25 largest cities in the United States[4] for 1930 was .48 ($T_{m25}=.46$; see note 3). The corresponding figure for the same cities for 1850 was .43 ($T_{m25}=.41$). For the 25 largest cities in Europe about 1931 the coefficient was .60.[5] In other words the largest cities of Europe are more nearly alike in population than are those of the United States; there has been a slight tendency toward greater uniformity in the United States since 1850.

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[3] A coefficient T_m which would represent "minimum measurable evenness" by a value of 0 may be found by the formula

$$T_m=2(\Sigma c-P)/P(N-1) \qquad \text{[Formula 1a]}$$

T_m is the ratio to B, the area of triangle I–IV–III ($B=\frac{1}{2}(PN-P)=\frac{1}{2}P(N-1)$), of D_m, the area between the distribution line and the lines I–IV and IV–III. $D_m=D-\frac{1}{2}P=\Sigma c-P$. Hence $D_m/B=2(\Sigma c-P)/P(N-1)=T_m$.

If the number of distributors is large T_m is practically the same as T. With few distributors there may be considerable difference. Thus, if there are only two distributors and the entire distribution is concentrated in one of them, $T=0.5$, $T_m=0$.

[4] The coefficients for the evenness of distributions of population in the United States given in this paper were computed from the U. S. Census reports.

[5] Computed from *Aperçu de la démographie des divers pays du monde* publié par l'Office Permanent de l'Institut International de Statistique, 1931, The Hague, 1932, pp. 42–43.

The coefficients T_{80} for the evenness of the distribution of rainfall[6] among the thirty years 1901–1930 for the following stations were: New York, .929; North Platte, .857; Yuma, .648. Comparison of the figures suggests differences in the variability of the annual rainfall at the three stations.

THE DISTRIBUTION OF RATIOS (COEFFICIENTS R)

In finding coefficient R (also R_a and R_{as}) we start with the assumption that the distributees and secondary distributors within each distributor are distributed with perfect evenness (or, in other words, show perfect association with one another) throughout all parts of the distributor. If, for example, the distributors are the areas of counties, we assume that the population in each county is evenly distributed over the surface of the county. If we are measuring the degree of association of colored and white population in a State in terms of counties, we assume that throughout all parts of any given county the ratio of colored people to whites is the same. On this basis any distributor may be represented by a right-angle triangle (Fig. 2,

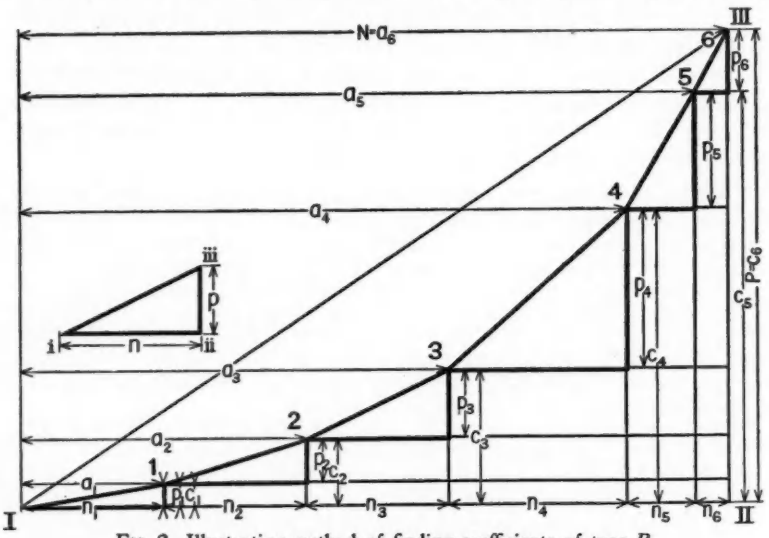

FIG. 2—Illustrating method of finding coefficients of type R.

inset) in which the length of the base i–ii stands for the number of secondary distributors (e.g., the area of the county in square miles) that the

[6] These and other coefficients on rainfall given in this paper are based on Clayton, H. H., World Weather Records. *Smithsonian Miscellaneous Collections*, Vol. 79, 1927; Vol. 90, 1934.

distributor contains, the height ii–iii for its population (or the number of distributees), and the steepness of the slope of the hypotenuse (line i–iii) for the ratio of population to secondary distributees (the density of population). By arranging the triangles for the entire distribution from left to right in increasing order of the densities (or ratios) that they represent, as in Figure 2 (main diagram), we obtain a graph in which the trend of the distribution line shows the degree of evenness or of association, much as in Figure 1. The coefficient R (also R_a and R_{as}) may be found either by measurements made upon such a graph or with the aid of the following formula (in which the numerator $= 2D$):

$$R = [n_1c_1 + n_2(c_1 + c_2) + n_3(c_2 + c_3) + \ldots]/PN, \quad \text{[Formula 2]}$$

n_1, n_2, n_3, etc., being the number of secondary distributors in the first, second, third, etc., distributors respectively when the latter are arranged in order of density of population; c_1, c_2, c_3, etc., being the cumulated populations in the corresponding distributors, and P and N being the total population and total number of secondary distributors respectively.

The evenness of the distribution, among the geographical divisions into which the U. S. Bureau of the Census divides the United States, of the per capita income of the farm population in 1929 was .84 (i.e., $R = .84$); that of the entire population, .78.[7] The high values of these coefficients show that there are no very great differences between the average incomes either of farmers or of the people as a whole when the geographical divisions are compared with one another.

The coefficients T for the evenness of distribution of the total production and consumption of coal among the European nations (exclusive of the U. S. S. R.) were .22 and .32 respectively in 1931.[8] On a per-capita basis the coefficients are higher, R being .38 and .64 respectively.[8] The latter figures indicate a disparity of about 10 to 6 as between the evenness with which the demand, or need, for coal is distributed among the peoples of the European nations and that with which the supply of coal is distributed. International trade, commercial rivalries, and wars are all results in large part of the disparity among the nations in regard to the control of and need for resources. Coefficients of this type may make clear the degree to which this disparity prevails with respect to particular resources.

A CRUDE METHOD OF FINDING T OR R

A short-cut method that yields approximate values for either T or R (also for T_a or R_a) may be applied to either areal or non-areal distributions. The coefficients found by this method may be designated K^* (i.e., "either T^* or R^*"). K^* is the mean of two factors, R' and R''.

[7] Computed from Leven, Maurice, H. G. Moulton, and Clark Warburton, *America's Capacity to Consume*, Washington, 1934, tables on pp. 172–173.

[8] Computed from statistics in *Statistical Year Book of League of Nations*, 1931/1932, Geneva, 1932.

The distributary is conceived as being divided into only two distributors, g_1 and g_2 (Fig. 3), in such a way that the ratio of population to secondary

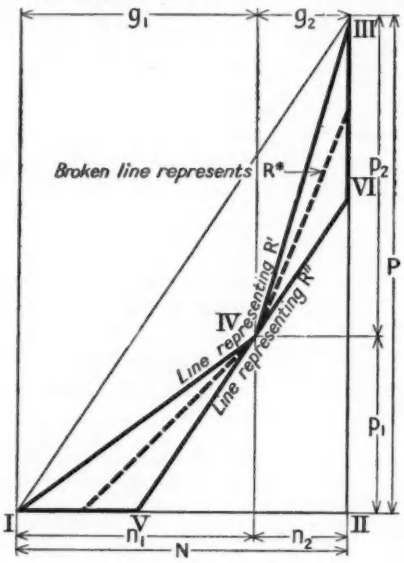

FIG. 3.

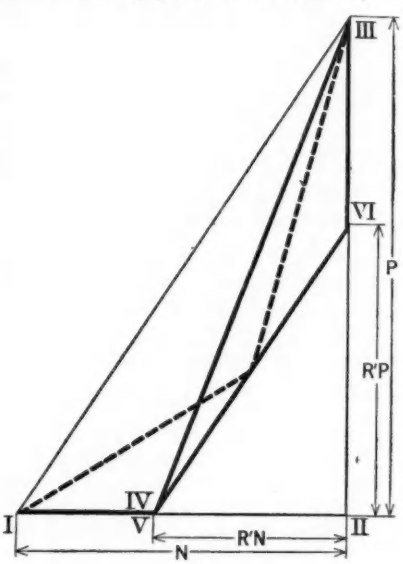

FIG. 4.

Figures 3 and 4 illustrate a method of finding K^* (i.e., either T^* or R^*). Fig. 4 shows that $R'' = R'^2$ (see footnote 9).

distributors in g_1 is less than P/N (i.e., the average ratio for the entire distribution). This being the case, the ratio in g_2 must exceed P/N. Let n_1 be the number of secondary distributors in g_1, and let p_1 and p_2 be the populations of g_1 and g_2 respectively. Hence n_2, or the number of secondary distributors in g_2, will be $N - n_1$. c_1, or the cumulated population in g_1, obviously is the same as p_1 and c_2 or the cumulated population in g_2 equals P. Thus, applying Formula 2, we obtain the following:

$$R' = [n_1 c_1 + n_2(c_1 + c_2)]/PN$$

or

$$R' = [n_1 p_1 + (N - n_1)\ (p_1 + P)]/PN = 1 - n_1/N + p_1/P \qquad \text{[Formula 3]}$$

In applying Formula 3 to an areal distribution, for example, p_1 is readily found by totalling the populations in the territorial divisions (States, counties, etc.) having less than the average density (P/N) and n_1 is found by totalling the areas of these units.

R' is an extremely easy factor to find, since the student does not have to perform the many problems in multiplication necessary in determining R, nor does he have to find the cumulated quantities necessary for both R and T. R', however, is based on the assumption that the populations within each of the two distributors, g_1 and g_2, are evenly distributed among the secondary distributors. This is not likely to be the case, and consequently R' tends to exaggerate the degree of evenness. This tendency may be corrected and a more probable value, K^*, found in the following manner.

Let R'' be a factor expressing the degree of evenness of a distribution in which the populations in g_1 and g_2 are distributed within each of these distributors as unevenly as possible. This situation, indicated by the distribution line I–V–VI–III in Figure 3, prevails when a part of g_1 has no population and the remainder of g_1 a density of P/N and when the density in the whole of g_2 is P/N except at one infinitely small point at which all the remaining population in the distribution is concentrated.

R' is the ratio of the area I–II–III–IV–I to the area of the triangle I–II–III. R'' is the ratio of the area of triangle V–II–VI to that of triangle I–II–III. It is not difficult to demonstrate that $R'' = R'^2$.[9] K^* is the value of evenness found on the assumption that the evenness of any actual distribution is exactly intermediate between R' and R''. Thus we take for our value of K^*[10] the mean of R' and R'':

$$K^* = \tfrac{1}{2}(R' + R'') = \tfrac{1}{2}(R' + R'^2)$$
$$K^* = \tfrac{1}{2}[1 - n_1/N + p_1/P + (1 - n_1/N + p_1/P)^2]$$

[Formula 4]

In the following table there are given comparable coefficients, T^*_{a100}, for Maine, Massachusetts, and Connecticut. These indicate the degree of evenness of the distribution of the human population and were based on the short-cut method (Formula 4). The figures are comparable because each State was first divided into 100 equal parts and the analysis based on these subdivisions (see p. 201).

[9] R' is the same irrespective of where the point IV happens to lie on line V–VI (Fig. 3). Assume that IV coincides with V (Fig. 4). The ratio of R' to R'' will therefore be the ratio of the area of triangle V–II–III to the area of triangle V–II–VI. The area of triangle V–II–III is $\tfrac{1}{2}R'NP$; that of triangle V–II–VI is $\tfrac{1}{2}R'N \cdot R'P$. The base of the latter triangle is obviously $R'N$; hence its height must be $R'P$, line V–VI being parallel to I–III since its slope, like that of I–III represents the ratio (density of population) P/N. Hence $R'' = R'^2$.

[10] The value of K^* is always within .125 of the actual coefficient K for the same distribution. That is to say, when K^* has a value of .375, K might be as high as .50 or as low as .25. As K^* approaches either 1 or 0 the differences become less. Thus if K^* has a value of .855, the value of K might be as high as .90 or as low as .81. If K^* has a value of .055, K might be anywhere between .10 and .01.

	1910	1930
Maine	.365	.36
Massachusetts	...	.31
Connecticut	...	.51

The next table shows trends in the evenness of the distribution of the population in the United States, New England, and Massachusetts as indicated by coefficients R_a*. The coefficients are based on statistics for the areas and populations of the States of the United States and of the counties of New England and Massachusetts and hence are not comparable for the three regions (see page 201, below).

	1790	1820	1830	1840	1850	1860	1870	1880	1890	1900	1910	1920	1930
Massachusetts:													
R_a* (counties)	.75	.71	.68	.63	.58	.54	.53	.51	.49	.48	.48	.48	.44
New England R_a* (counties)						.44	.46	.40	.34	.33	.31	.29	.27
United States: R_a* (States)					.222	.342	.360	.410	.467	.490	.526	.538	.549

THE FACTOR OF RELATIVE CONCENTRATION, C

In Figure 5 four distribution lines are shown for four different distributions. All four have a coefficient K of .5 (as the following discussion applies equally to coefficients of the T or R types the letter K should be understood as meaning "either T, T_a, R, or R_a"). In the distribution represented by Line 1 (I–V–III) 50 per cent of the distributors (i.e., $(1-K)N$ distributors) have no population and the entire population is evenly distributed among the remaining 50 per cent (i.e., KN) of the distributors. A distribution of this type may be called "a distribution of 50 per cent evenness with maximum emptiness." In the distribution represented by Line 2 (I–VI–III) 50 per cent of the population (i.e., $(1-K)P$) is concentrated at one mathematical point and the remainder of the population (i.e., KP) is evenly distributed among all the distributors. A distribution of this type may be called "a distribution of 50 per cent evenness with maximum concentration." Line 3 (I–VII–III) represents a type of distribution exactly intermediate between the types represented by Lines 1 and 2. This type might be called "an average distribution of 50 per cent evenness." The degree to which any actual distribution departs from the average type and approaches either one of the extremes represented by Lines 1 or 2 may be determined and expressed by a supplementary "factor of relative concentration," designated by the letter C. This factor is 0 for a distribution of

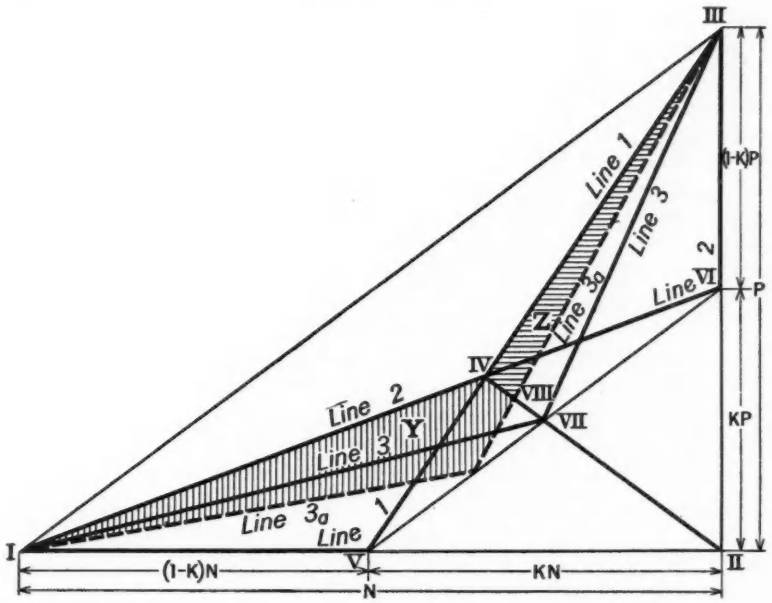

FIG. 5—Illustrating method of finding C, the factor of relative concentration.

type 3, +1.0 for a distribution of type 2 and −1.0 for a distribution of type 1.

The factor of relative concentration is found in the following manner: as in Figure 5 distribution lines are plotted for the actual distribution (illustrated by Line 3a; dashed line) and for the distributions having the same coefficient K as that of the actual distribution with maximum "emptiness" (Line 1) and with maximum concentration (Line 2). Line 1 will always coincide with I–II from I to a point V at a distance of $(1-K)N$ from I and thence run directly to III. Line 2 will always coincide with III–II from III to point VI at a distance of $(1-K)P$ from III and thence run directly to I. A line is then drawn from point IV at the intersection of Lines 1 and 2 to point II. The areas marked Y (vertical ruling) and Z (horizontal ruling) are then measured, either graphically or with a planimeter.

$$C = 1 - 2Y/(Y+Z) \qquad \text{[Formula 5]}$$

It is obvious that if $Y = Z$ the distribution is of the average type and C will be 0. The area of Y becomes smaller with increasing relative concentration

until Y disappears when maximum relative concentration is reached. C correspondingly becomes larger until it reaches $+1$. Similarly Z becomes smaller with decreasing relative concentration until it disappears with maximum "emptiness," in which case C becomes -1. (C may be found mathematically without the aid of a graph by a somewhat complex method that cannot be explained here.)

The significance of C may be illustrated thus. T_{a100}, the coefficient of the evenness of the distribution of population in Maine for 1930, was .29; C was $-.29$. If C had been -1.0 this would have meant that .71 $(1-T_a)$ of the area of the State was uninhabited and the entire population evenly distributed over the remaining .29 (T_a) of the area. If, on the other hand, C had been $+1.0$ it would have meant that .29 of the population was evenly distributed over the entire measurable area of the State and the remaining .71 of the population concentrated at one mathematical point. If C had been 0 there would have been an intermediate condition between these two extremes. T_{a100}, being as low as .29, in itself shows a fairly high degree of unevenness. C being $-.29$ indicates that this unevenness was due rather to the existence of extensive areas of very sparsely settled or uninhabited country than to the excessive concentration of a relatively large part of the population in a small area, accompanied by a generally even distribution of the remaining population over the rest of the state.

A value of C that is 0 or close to 0 may be regarded as representing a higher degree of evenness in the distribution as a whole than that which prevails when C approaches $+1.0$ or -1.0. This suggests that, having determined the values of K and C for any distribution, we should, if we wish to obtain a more refined coefficient of evenness, reduce K somewhat so as to bring out the degree of unevenness indicated by C. Such a refined coefficient K may be designated K_4 (i.e., T_4 or R_4 as the case may be).[11]

[11] Coefficient K in the statisticians' terminology might be called an inverse coefficient of dispersion; $1-K$ would be a coefficient of dispersion. The value of any coefficient K is influenced by the degree to which the variates (distributees) on either side of the mean differ among themselves. K, however, is not affected by the *skewness* of the variates, i.e., the degree to which the variates themselves "tend to be dispersed more on one side of the mean than on the other" (W. L. Crum and A. C. Patton, *An Introduction to Economic Statistics*, New York, 1925, p. 211). Factor C indicates the skewness. In factor K_4 both degree of dispersion and skewness are taken into account.

Two other methods of finding coefficients, K_b and K_c, that are equivalent to K_4 (although they do not yield the same values except with maximum and minimum evenness) may be suggested. The first is to find σ, the standard deviation of the variates from the mean, and σ_m, the standard deviation that would prevail with maximum unevenness. In this case the coefficient of evenness, K_b, would be $1-\sigma/\sigma_m$. The second method is to find the actual sum of the squares of the variates Σp^2 and

Obviously when C is 0, K should not be changed and K_t should equal K. It is suggested that the area of triangle U bounded by lines I–V–VI be taken as a criterion for fixing the maximum amount by which K should be reduced when $C = \pm 1.0$. As K either increases from 0.5 to 1.0 or diminishes from 0.5 to 0, area U decreases to zero. If we designate the ratio of U to the area of triangle I–II–III (i.e., $2U/PN$) by the letter Q, the refined coefficient would then be $K_t = K \pm CQ$, CQ being added to K when C is a minus quantity and subtracted from K when C is a plus quantity.

That $Q = K - K^2$ may be shown thus: U is the area of triangle I–V–VI (which is $\frac{1}{2}KPN$) minus the area of triangle V–II–VI, which is $\frac{1}{2}KP \cdot KN$. Hence $U = \frac{1}{2}(K - K^2)PN$, and $Q = 2U/PN = K - K^2$. Consequently:

$$K_t = K \pm C\,(K - K^2) \qquad \text{[Formula 6]}$$

The values of T_∞ for the evenness of distribution of rainfall over a period of 30 years are, as we have seen, 0.857 for North Platte and 0.648 for Yuma. The values of C for these two stations are -0.124 and -0.03 respectively. Applying Formula 6 we find that $T_{t\infty}$ for North Platte was 0.842 and for Yuma 0.641.

C would have been -1.0 for North Platte if the entire rain that fell during the 30 years had been evenly distributed during 0.857 of the 30 years and no rain had fallen during the remaining 0.143 of the years. On the other hand C would have been $+1.0$ if 0.857 of the rainfall had been evenly distributed during the entire 30-year period and the remainder had fallen at one instant of time. Since the actual factor C for North Platte is -0.124, a distribution that tends slightly toward the extreme with dry years is suggested.

Coefficient T_∞ representing the evenness of the distribution among the 30 years 1901–1930 of the rainfall at New York City is 0.929. The corresponding figures for the months of July, August, and September of the period 1901–1930 are considerably lower:

1901–1930	T_∞	$T_{t\infty}$	C	*Mean for month*
July	.70	.65	−.23	4.17 in.
August	.69	.61	−.36	4.34 in.
September	.74	.73	−.038	3.18 in.

the sums of the squares of the variates with minimum evenness, P^2, and with maximum evenness, $N(P/N)^2 = P^2/N$, P being the number of distributees and N the number of secondary distributors. In this case K_c is $N(P^2 - \Sigma p^2)/P^2(N-1)$.

The writer believes that coefficient K is easier to determine than either of these, because K does not necessitate the laborious process of squaring the variates. Coefficients K_b and K_c, it is true, take into account the element of skewness, leading one directly to a value that is comparable to K_t, whereas in the case of K one is obliged to determine first C, which indicates the skewness, and then K_t. In one sense this might be regarded as an advantage of K_b or K_c over K. On the other hand, C discloses the characteristic of skewness, which is of interest in itself.

The statisticians' coefficient of variation, which is the standard deviation divided by the mean (*op. cit.*, p. 194) might be used as a measure of unevenness, but it may yield values that are greater than 1 (or 100 per cent).

The similarity of the values for T_{30} shows that, as between the three months, the chances, in any particular year, that the rainfall would be the same as the 30-year mean for the month were about equal. Further study would be needed to arrive at a sound explanation for the differences in the values of C. The low values of C for July and August indicate that irregularity of the July and August rainfall over the period of 30 years was due rather more to a relatively high frequency of dry Julys and Augusts than to excess of rainfall during a relatively few wet Julys and Augusts. It is possible that the higher value for C for September as compared with July and August may be partly accounted for by excessive rains during certain years, as a result of hurricanes.

When applied to the analysis of the distribution of mean monthly rainfalls among the months of the year, the coefficient T and the factor C indicate characteristics of the distribution that might conceivably be used in the classification of rainfall types. Low values of T suggest a long dry season and a short rainy season or possibly two long dry and two short rainy seasons.[12] Medium values of T when accompanied by large minus values of C suggest rainy and dry seasons of approximately equal length, with extreme drought during the dry season. Medium values of T accompanied by large plus values of C suggest even distribution of rain during most of the year except for a peak confined to a short period. The following figures bring out some of the differences for stations having contrasted rainfall regimes:

	T_{12}	$T_{\frac{1}{12}}$	C	*Mean annual rainfall*
New York	.81	.81	−.03	40.3 in.
Valentia, Ireland	.88	.87	+.82	57.4 in.
Athens	.61	.39	−.90	15.2 in.
Catania, Sicily	.58	.40	−.74	33.3 in.
Freetown, Sierra Leone	.34	.19	−.68	138.6 in.

LOCATIONAL DISTRIBUTIONS

LINEAR DISTRIBUTIONS (COEFFICIENT L)

The following technique is suggested for determining the degree of evenness with which distributees are distributed along a line (the distributary).

[12] Coefficient T, of course, takes no account of the relative positions of the distributors. If a station has a mean annual rainfall of, say, 30 inches and this is distributed evenly among three months of the year, 10 inches to each month, T would be the same whether the rainy months were consecutive or separated from one another by one or more rainless months. The writer has developed a method of measuring the evenness of distributions in time, in which the relative positions of the distributors (days, months, or years, etc.) are taken into account. This technique, based on the technique for analyzing linear distributions discussed in the next section, cannot be explained in the present paper owing to limitations of space.

The method could, for example, be used in measuring the evenness of the distribution of railroad stations along a railroad or of population along a strip of territory of a uniform width paralleling the railroad. In this case maximum evenness obtains when the intervals between the several distributees and also the two intervals between the end distributees and the ends of the line are all equal. Minimum theoretical evenness (or maximum unevenness) obtains when all the distributees are concentrated at either extremity of the line.

Let us call the line with respect to which a distribution is to be analyzed Line A. The degree of evenness is measured with the aid of an adaptation of the Lorenz curve. A rectangle is constructed whose length represents Line A and whose height represents the total number of distributees. For any actual distribution a distribution line is drawn across the rectangle connecting the lower left-hand with the upper right-hand corner. This distribution line must be drawn so that its distance from the base (or lower side) of the rectangle at any point indicates the number of distributees to the left of the corresponding point on Line A. If, for example, a point on the distribution line lies one quarter of the way from the left to the right side and three quarters of the way from the base to the top of the rectangle, it means that three quarters of the distributees are on the quarter of Line A farthest to the left. With perfect evenness of distribution the distribution line would form a diagonal connecting the lower left with the upper right corners of the rectangle. Hence the degree to which any actual distribution line departs from this diagonal is a measure, M, of the degree of linear *unevenness*. The degree of departure is measured in terms of the area between the distribution line and the diagonal. With maximum unevenness the distribution line would coincide either with the base and right-hand side of the rectangle (if all the distributees were at the right-hand end of Line A) or with the left-hand side and top of the rectagle (if they were all at the left-hand end). Hence, with maximum unevenness the area separating the distribution line and the diagonal would be half the area of the rectangle, or $\frac{1}{2}PN$, when P is the number of distributees and N the length of Line A. The degree of unevenness may then be represented by the ratio of factor M to $\frac{1}{2}PN$, or $2M/PN$, and the degree of evenness (coefficient L) by

$$1 - 2M/PN \qquad \text{[Formula 7]}$$

This coefficient may be found either graphically or mathematically. If the exact location on the line of every individual distributee cannot be determined, the line is divided into distributors and coefficient L found on the assumption that perfect evenness prevails within each distributor. If the distributors are of equal lengths, a simple technique is to cumulate the popu-

lations in the distributors from left to right, finding values c_1, c_2, c_3, etc. for the successive distributors much as in the case of coefficients of the T type, except that here the distributors are cumulated in order of position on the line and not in order of population. For each distributor values v_1, v_2, v_3, etc., are also found showing the cumulated number of distributees for each distributor were the population arranged with maximum evenness. The difference between the value of c and the value of v is then taken for each distributor, and M is the sum of the differences.

The lines bounding Figure 1 may serve to illustrate a rectangle of the type employed in analyzing a linear distribution. In this case the distributors represented by the vertical columns should be conceived as arranged in the order of their positions along Line A. With perfect linear evenness lines a_1, a_2, a_3 etc. would meet lines c_1, c_2, c_3 etc. on the diagonal I–III (a/N would always equal c/P). The distribution line for an actual distribution may lie either entirely below the diagonal (as in Fig. 1), entirely above it, or it may cross the diagonal. If the distributors are of unequal lengths each value of c and of v must be multiplied by n, the length of the corresponding distributor (see Fig. 2). M is the sum of the differences between the products cn and vn for the several distributors.

The degrees of linear association of the distributions of two elements A and B along a Line A may be found as follows. A rectangle is constructed, of which the width represents the length of Line A and the height the total number of distributees comprised in element A and also the total number comprised in element B. A distribution line is drawn accordingly for each element. The area M_{as} separating the two distribution lines is a measure of the degree of disassociation. With perfect linear association the two distribution lines would coincide and M_{as} would be 0; with maximum disassociation—that is, if element A were concentrated at one end and element B at the other end of Line A—M_{as} would be the area of the rectangle, or PN, P being the height and N the length of the rectangle. L_{as}, or the coefficient of linear association, would therefore be:

$$1 - M_{as}/PN \qquad \text{[Formula 8]}$$

If the analysis is made in terms of N subdivisions of Line A of equal lengths, M_{as} may be found without the aid of a graph. For each subdivision in order from left to right along Line A two values, c_a and c_b, are found, i.e., the cumulated number of distributees of elements A and B respectively. The several values of c_b are then multiplied by P_a/P_b in order to convert them into quantities that will total up to P_a, or the total number of distributees of element A, P_b being the total number of distributees of element B. M_{as} is the sum of the differences between the several values of c_a and the several values of $c_b\ P_a/P_b$, and the coefficient L_{as} is

$$1 - M_{as}/P_aN \qquad \text{[Formula 8a]}$$

AREAL DISTRIBUTIONS

A few more terms must be defined in order that the special problems relating to areal distributions may be discussed concisely. If we wish to analyze either the evenness or the association of areal distributions we must ordinarily break up the regions within which the distributions lie into smaller *areal subdivisions.* Theoretically it is possible to subdivide any region into areal subdivisions in an infinite variety of different ways, and it is this that complicates the problem. Any actual combination of subdivisions into which a region is or might be subdivided will be called a *system of areal subdivisions.* The areas of the States constitute one system of areal subdivisions of the territory of the United States, the areas of the counties another, tenths of the total area still another, etc. If the areas of the subdivisions are all of different sizes (as in the case of States or counties) the system is said to be *irregular;* if the areas are equal in size (comprising, for example, tenths or hundredths of the whole region) the system is *regular.* If a system consists of only a few subdivisions each of which comprises a relatively large part of the region, the *relative scale* of the system and of any coefficient based upon it, is *coarse.* When many relatively small subdivisions are employed, the scale is *fine.* The scale of a regular system may be expressed numerically. If, for example, there are 10 equal subdivisions, the scale is one-tenth, if there are 100 the scale is one one-hundredth (thus T_{a100} means an areal coefficient of evenness based on a regular system of areal subdivisions of scale 1/100).

When we measure the degree of areal evenness of a distribution the areal subdivisions on which we base our calculations constitute areal distributors. It is important to note, however, that when we measure the degree of areal association of two distributions the areal subdivisions, while they serve to break up the distributary into distributors and thereby to determine what the distributors shall be, are not themselves the distributors. The distributary in this case is not the region but one of the two elements whose areal association we wish to measure, and any areal subdivisions that contain no part of the distributary exert no effect whatever upon the coefficient R_{as}.

The Evenness of Areal Distributions[13]

Absolute and Relative Areal Evenness

Let us draw a map of a square township showing the location of every house. The houses are distributed over the surface of the township with

[13] In a publication entitled "Density and Distribution of Population in the United States at the Eleventh Census" (*American Economic Association, Economic Studies,* Vol. 2, No. 6, December, 1897) Professor W. F. Willcox presents a table

a certain degree of evenness. If they are all clustered in one corner of the township this degree of evenness is close to the minimum. If they are clustered together near the center of the township there is a somewhat higher degree of evenness; if they are evenly spaced, forming a smooth pattern that covers the whole area of the township, the degree of evenness is at the maximum. The degree of areal evenness represented by any one of these or other arrangements of the houses within the township may be called the degree of *absolute areal evenness*. The degree of absolute areal evenness is a definite and fixed characteristic of an areal distribution and is quite independent of any particular system of areal distributors into which the distributary (township) might be divided.

A simple areal coefficient T_a or R_a, however, is based on only one system of distributors, and consequently does not show the degree of absolute evenness. The simple coefficients show merely the degree of evenness in relation to a particular system of distributors, and hence they will be called *coefficients of relative areal evenness*. Their values (magnitudes) are affected by the degree of absolute evenness but they also may be largely influenced by the characteristics of the particular system of distributors with reference to which they are calculated, as will now be shown.

Areal Coefficients T_a and R_a

The relative coefficients T_a and R_a disclose certain important characteristics of a distribution. What these characteristics are depends in part upon the scale of the system of areal distributors employed and in part upon the manner in which the boundary lines between the several areal distributors happen to subdivide both the area (distributary) and the population (distribution).

The effects of differences in scale upon different coefficients may be illustrated by the simple examples shown in Figures 6 and 7 and also, in greater detail, in Figure 8. In Figure 8 a square township (distributary) is divided into two different regular systems of areal distributors. System 1

of figures (p. 395) that aims to show the degree of unevenness with which each State was populated in 1890. The density of population of each county in a State was found; then the average difference between the figures for the counties and the density of population in the State as a whole. This average difference is expressed as a percentage of the average density in the State. California exhibits the greatest unevenness, with 1550 per cent; Indian Territory, Vermont, and Iowa have the lowest percentages, 26, 28, and 31 respectively. Since no account is taken of the relative areas of the counties, several very small but densely peopled counties might exert a disproportionate effect on the percentages. Furthermore, no maximum degree of unevenness is fixed by this method.

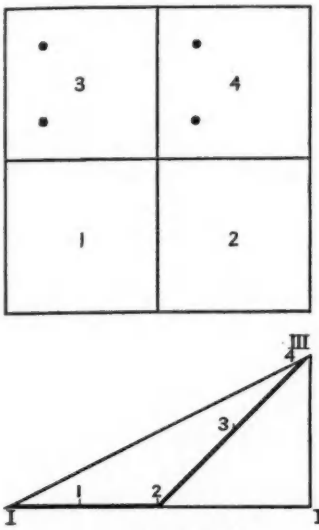

FIG. 6

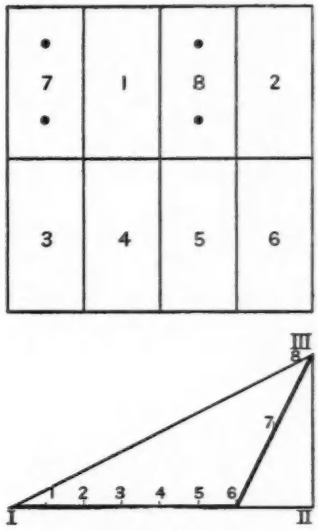

FIG. 7

FIGS. 6 and 7—The manner in which the distribution line and the coefficient T_a (or R_a) representing an areal distribution are affected by the number of distributors. The four dots are distributed with respect to the outer boundaries of the squares in identically the same manner in Figure 6 as in Figure 7; the distribution line in Figure 6, however, indicates a coefficient T_a of .50, whereas the distribution line in Figure 7 shows a coefficient of only .25.

(scale 1/4) consists of four distributors each of which comprises exactly one quarter of the township (their boundaries are shown by the heavy lines in Fig. 8). System 2 (scale 1/64) consists of sixty-four equal distributors (bounded by light lines in Fig. 8). Each distributor of System 1 contains sixteen distributors of System 2. Now let us see what the values of coefficients T_{a4} and T_{a64} are with the different arrangements of the population shown in Figure 8.

(1) Population entirely concentrated within one distributor of System 2 (this distributor might be anywhere in the township): $T_{a4} = 1/4 = .25$; $T_{a64} = 1/64 = .015625$.

(2) Population evenly distributed over the entire area of one distributor of System 1 and therefore evenly distributed among sixteen distributors of System 2: both T_{a4} and $T_{a64} = 1/4$.

(3) Population evenly distributed among four distributors of System 2, all 4 of these being included within one distributor of System 1: $T_{a4} = 1/4$; $T_{a64} = 1/16$.

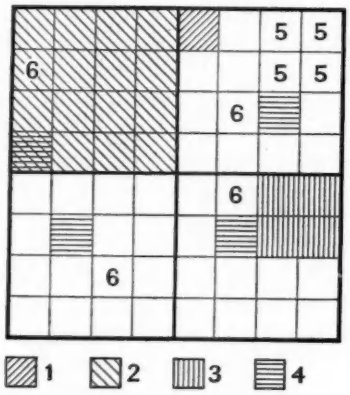

FIG. 8.

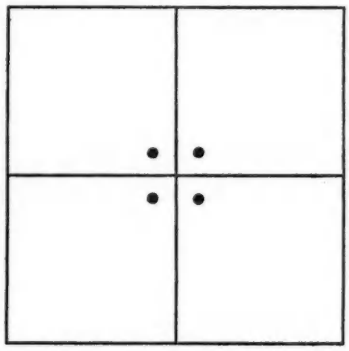

FIG. 9.

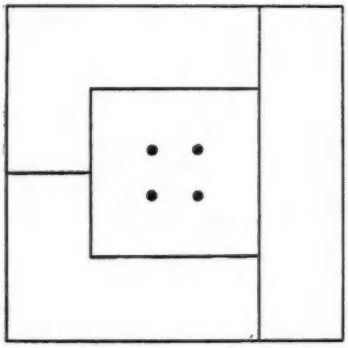

FIG. 10.

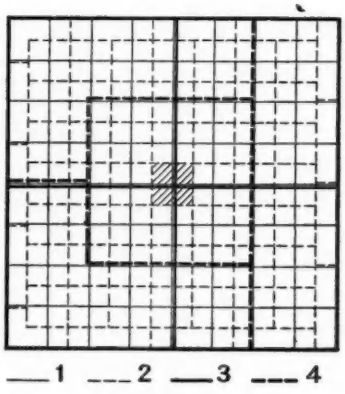

FIG. 11.

FIG. 8—The relationship of six areal distributions to two different systems of areal distributors.

FIGS. 9 and 10—How a coefficient of evenness may be affected by the location of the boundaries of the distributors. In both diagrams the four dots are distributed in the same manner with respect to the outer boundaries of the squares and there are four distributors of equal area; T_a with respect to Figure 9 is 1.0; with respect to Figure 10 only .25.

FIG. 11—The relationship of a distribution to different systems of distributors.

(4) Population evenly distributed among four distributors of System 2, one in each of the four distributors of System 1: $T_{a4} = 1$; $T_{a64} = 1/16$.

(5) Population evenly distributed among sixty distributors of System 2,

there being four empty distributors (shown by the "5's" on Fig. 8) of System 2 in one of the distributors of System 1: $T_{a4} = 60/64$; $T_{a64} = 60/64$.

(6) Population evenly distributed among sixty distributors of System 2, there being one empty distributor of System 1 in each of the four distributors of System 2 (shown by the "6's" on Fig. 8). $T_{a4} = 1$; $T_{a64} = 60/64$.

Study of the above should bring out the following facts:

(1) If a fine-scale coefficient has a high value, any coarse-scale coefficient for the same distribution will also have a high value (Nos. 5 and 6). A high value of evenness on the part of a fine-scale coefficient means that there is a high degree of absolute areal evenness.

(2) If a fine-scale coefficient has a low value, coarse-scale coefficients for the same distribution might have either high (No. 4) or low (No. 3) values, depending upon how the coarse-scale distributors subdivide the population.

(3) If a coarse-scale coefficient has a low value, no fine-scale coefficient for the same distribution can have a higher value, although the value of any fine-scale coefficient might vary considerably, depending upon whether the population is dispersed or concentrated within the limits of the coarse-scale distributors (Nos. 1 and 2).

Generally speaking, therefore, a fine-scale coefficient indicates more in regard to the detailed texture of a distribution pattern and a coarse-scale coefficient shows more in regard to the general evenness of the distribution of the population over large parts of the distributary region; and each coefficient may fail completely to bring out the characteristics of the distribution shown by the other.

A series of fine-scale coefficients R_a for successive census years showing the evenness of the distribution of the population of the United States among counties as distributors would probably disclose an increasing tendency toward unevenness as a result of the more rapid increase of the urban than of the rural population; on the other hand, a series of coarse-scale coefficients showing the evenness with respect to States or larger geographical divisions would disclose increasing evenness..

The value (magnitude) of any relative areal coefficient, T_a or R_a, is also influenced by the manner in which the boundary lines between the distributors are drawn (Figs. 9 and 10). If, for example, we were to divide the territory of the United States into only two areal distributors by a line running northeast and southwest and find T_{a2} with respect to these, the coefficient would indicate a much higher degree of evenness than would be the case if the line were drawn from north to south. The coarser the scale, the greater will the differences tend to be between the values of two or more coefficients determined with reference to systems of distributors of the same

scale but with different boundary lines. When the scales are fine the differences are much less likely to be pronounced.

This may be illustrated as follows. Let a distributary region first be divided into two regular systems (scale 1/64) each consisting of sixty-four equal distributors (the boundaries of the four systems of distributors discussed in this paragraph are indicated by the different kinds of lines on Figure 11). Let the subdivisions be made in such a way that, so far as possible, each distributor of System 1 comprises one quarter of the area of each of four distributors of System 2, and vice versa. Let the entire population be evenly distributed over the whole surface of one of the distributors of System 2. This means that the population will be equally divided among four distributors of System 1 and therefore evenly distributed among (although, of course, not evenly distributed over the whole surface of) these four distributors. Coefficient T_{a64} measured with respect to System 2 will be 1/64; measured with respect to System 1 it is 4/64. The difference between these two factors is only 3/64. Now let us divide the same region into two regular systems of distributors (Systems 3 and 4) each of scale 1/4, in such a way that the 1/64 of the total area on which the population is located is divided equally among four distributors of System 3, but lies wholly within one distributor of System 4. In this case T_{a4} measured with respect to System 3 will be 1.0 but measured with respect to System 4 it will be ¼ and the difference between the two factors will be ¾, or sixteen times as much as it was when the analysis was made with reference to the fine-scale distributors. Theoretically it is conceivable that a population might be evenly distributed checkerboard fashion among alternate square distributors of a fine-scale system (System 1), in which case T_a would approach 1/2. By placing the corners of the distributors of a second system (System 2) at the centers of the distributors of System 1, we should then find that the population was evenly distributed among the distributors of System 2, giving a coefficient of approximately 1. It is, however, highly improbable that such a symmetrical arrangement of an areal distribution could occur in nature. Under actual conditions one system of distributors will, of course, break up among two or more distributors certain clusters of population that are entirely included in single distributors of another system. While this may increase the tendency toward evenness in so far as these particular clusters are concerned, the effect will be largely balanced by the opposite effect in the case of other clusters.

An example may be given that illustrates for the same region the differences between actual coefficients T_a when calculated with respect to two different systems of distributors of the same scale. The present area of the United States was first divided into a system of twenty equal distributors (by the "method of combining territorial units" described below, on page 202) and T_{a20} for the distribution of population was found with respect to these distributors for 1790[14] and for 1930. The country was then redivided into a second system of twenty equal distributors demarcated in such a way that their boundaries, so far as possible, subdivided each distributor of the first system into two equal parts and the values of T_{a20} were found with respect to this second system. The values for 1790 were .115 and .129 ($\overline{T}_{a20}$, the

[14] In calculating the figure for 1790 the area west of the Mississippi was assumed to be uninhabited.

mean, = .121) and for 1930 .439 and .467 ($\overline{T}_{a20} = .453$). In both cases the difference was less than two per cent, and yet distributors of not very fine scale were employed.

Comparability of Coefficients T_a and R_a[15]

Since, therefore, the value (magnitude) of any areal coefficient T_a or R_a is influenced to a greater or less degree by the characteristics of the system of distributors with reference to which it is computed, it follows that no two of these coefficients are comparable with one another unless both are computed with reference to comparable systems. Irregular systems are never comparable, and hence no two coefficients R_a for distributions in different regions are ever comparable. For this reason one should not compare R_a showing the degree of evenness of the distribution of population among the areas of the States of the United States with R_a determined with reference to the areas of the nations of Europe.

It may be assumed, however, that two areal coefficients for distributions in different regions are approximately comparable if both coefficients are determined with reference to regular systems of distributors of the same scale, provided the scale is not too coarse. Thus, a coefficient T_{a100} indicating the evenness of the distribution of the population of the United States among equal areas each of which comprises 1/100 of the area of the United States may reasonably be compared with a coefficient T_{a100} found in a similar manner for Europe.

Two methods may be suggested for dividing a region into a given number of distributors of equal areas and of finding the population in each distributor.

Method of squares. If N is the number of distributors into which the region is to be divided and A the area of the region, A/N is the area of each distributor. On a base map of the region showing the boundaries of the territorial units (such as States, counties, etc.) for which population statistics are to be used in the analysis, draw a grid of rectangular coordinates with lines at intervals of $\sqrt{A/N}$. The

[15] A short-cut method may be suggested for finding the degree of evenness of areal distributions for a single region, provided the purpose is only to compare different types of distribution in one region and not to make comparisons between distributions in two or more different regions. This is simply to assume that all of the territorial units (such as counties or minor civil divisions) on which the analysis is based are of equal area. One may then regard each unit as a distributor and find the simple coefficient T, for the evenness of distribution among these units, coefficient T being an easier factor to find than coefficient R_a. This method gives a fair approximation of the areal evenness, as long as the territorial units do not differ greatly in size. It could hardly be applied to the United States, using States as the distributors, but it yields reasonable values for many of the individual States on the basis of a subdivision into counties or minor civil divisions. For New Hampshire the evenness of distribution of the population among the minor civil divisions in 1910 was .33; in 1930, .30.

squares (distributors) marked out by these coördinates will then each have an area of A/N. The areas of the parts of the region comprised within squares through which the boundary of the region passes must be measured or estimated, and these areas arbitrarily combined so as to form areas (distributors) each of approximately A/N. If the population in each territorial unit is assumed to lie at the center of the unit and is so plotted on the map, the population in each distributor may readily be found by totalling.

The scale of a system of areal subdivisions should be so selected that the area of each subdivision is larger than the area of the largest territorial unit for which population statistics are used.

Method of combining territorial units. On a piece of coördinate paper draw a diagonal line that represents the total area A in such a way that one set of ordinates divides the line into N parts (or multiples thereof), each part representing a distributor. Along this line mark off subdivisions representing the territorial units for which statistics are employed, arranging them in geographical order (i.e., placing subdivisions representing adjacent units next to one another) and making the length of each subdivision proportional to the area of the unit for which it stands. Write the names of the units next to the corresponding subdivisions. From this diagram one may read off the units that fall wholly within each distributor and readily determine the fraction of any unit that should be assigned to any distributor. Thus one distributor might be seen to contain New England, New York, and two-thirds of Pennsylvania; the second distributor, one-third of Pennsylvania, Ohio, etc. The assumption may be made that if a distributor includes a certain fraction of the area of a unit it also includes the same fraction of the total population in that unit, or else if considerable precision is desired, the boundaries of the distributors may be marked on a map and the population in each accurately found. (For example, if a distributor must include one third of Iowa, adjacent to Missouri, mark off such an area on a base map of Iowa and from county statistics determine its population.)

T_{a100} for Maine for 1910 was .299 and for 1930,[16] as we have seen, .290 per cent. Although the population of Maine increased between 1910 and 1930, the small amount of the differences between these factors for the two years indicates that the rate of change must have been fairly uniform over the whole surface of the State.

If we wish to obtain comparable coefficients that will show the degree of evenness of areal distributions in different regions with reference to relatively large parts of the regions, we cannot simply compare two coefficients T_a of coarse scale, because, as already explained, the values of such coefficients are largely affected by the way in which the boundaries of the distributors are drawn. In order to get around this difficulty compound relative areal coefficients may be employed. Such coefficients will be designated $\overline{T}_a$ and may be determined as follows.

If we divide a distributary region into two or more different regular

[16] These coefficients are based on the subdivisions of the area of Maine into 100 equal parts by the method of squares.

systems of distributors all of the same coarse scale but with widely divergent boundary lines, the values of the several coefficients T_a determined with respect to these systems are likely to differ to a considerable degree (unless there is a high degree of absolute evenness). By taking the mean of the several values we may iron out the differences, as it were, and thus obtain a compound coefficient $\overline{T}_a$ that approaches a fixed and definite value and that may be regarded as approximately comparable with other compound coefficients $\overline{T}_a$ found in the same manner for distributions in other regions.

The procedure suggested could be carried out in many different ways. Obviously, the greater the number and variety of the factors T_a, the greater should be the precision of their mean value, $\overline{T}_a$. The following, for example, is tentatively suggested as a method that should yield satisfactory results in most cases. The distributary region is divided into four regular systems of distributors. (How this division could be made for a circular distributary region is illustrated in Figure 12, where the boundaries between

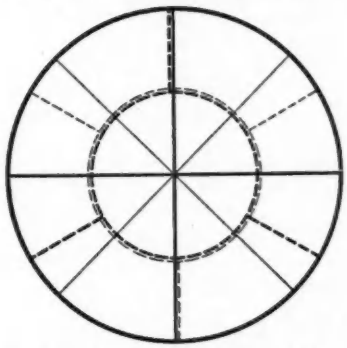

FIG. 12—Illustrating a possible method of subdividing a circular region into four different systems of distributors, all of the same area, as a basis for calculating a compound coefficient $\overline{T}_{a4}$.

the distributors of the four systems are distinguished by different kinds of lines.) In System 1 the four distributors are divided from one another by four straight lines radiating from the center of the region and, of course, drawn in such a way that all four distributors are of equal area (heavy unbroken lines). In System 2 the boundary lines between the four distributors are drawn in a similar manner except that each line divides the distributors of System 1 into two equal parts (light unbroken lines). In System 3 (heavy broken lines) one distributor comprises the part of the region within a circle centering on the center of the region and drawn so as to include ¼ of the area of the distributary within its circumference. The other three

distributors of System 3 each comprises ⅓ of the part of the region that lies outside the circle, the boundaries between them being formed by parts of radii drawn from the center of the region. In System 4 (light broken lines) the central distributor is the same as that of System 3; the boundaries between the three outer distributors, however, divide the areas of each of the outer distributors of System 3 into two equal parts. T_{a_4} is determined with respect to each system and the mean of the 4 values of T_{a_4} is $\overline{T}_{a_4}$.

Any one of these four factors T_a could approach or attain a value of 1 without necessarily indicating a high degree of evenness. If, however, all four factors are in the neighborhood of 1, a high degree of evenness must prevail with respect to the distribution of the population among large parts of the area.

To divide a non-circular region as described above is somewhat laborious but may be done with the aid of transparent polar coördinate paper as follows: (1) Having found on a map the center of the region (see pp. 206–207), place the center of the coördinates over the center of the region and trace the boundaries of the region on the coördinate paper. (2) Find the area of the region, as so traced, in square millimeters. (3) Find the area of each of the circles (πr^2) shown on the coördinate paper in terms of square millimeters and inscribe this quantity at the point were the circumference of each circle crosses one of the radial ordinates. (4) For each circle find the area of each of the sectors into which it is subdivided ($\pi r^2/n$, n being the number of sectors) and indicate this on the coördinate paper. (5) Any one of the radial ordinates may be taken as the boundary between two distributors (Nos. 1 and 2) of System 1. Beginning at the point where the boundary of the region intersects this ordinate and working along the boundary, cumulate the areas of the parts of the sectors *within the region* until they amount to ⅛ of the total area. This will locate the radial ordinate marking the boundary between two distributors of System 2. The point where the cumulated areas amount to ¼ of the total area will locate the ordinate bounding two distributors (Nos. 2 and 3) of System 1, and so on. (6) The central circular distributor of Systems 3 and 4 may be established by drawing a circle of a quarter of the total area around the center, and the boundaries of the three peripheral distributors of System 3 and 4 may then be found in much the same way as are those of the distributors of Systems 1 and 2. If, however, the boundary of the region cuts through a circle of ¼ of the total area described around the center of the region, the part of this circle within the region will have an area of less than ¼ of the total area. If this be the case the central circle must be drawn in such a way that it includes ¼ of the area. This may be done by finding the area of the smallest circle (around the center) that lies wholly within the region and then cumulating outward the areas of the parts of the region included in the successive circular bands surrounding this smallest circle until the area of the smallest circle plus the cumulated area of the bands amounts to ¼ of the total area.

Approximate Coefficients of Absolute Areal Evenness

As yet the writer has been unable to devise a technique for finding a

definitive coefficient indicating the degree of absolute areal evenness—a coefficient not dependent to some extent on the systems of areal distributors employed in the analysis. Such a coefficient might be designated by the letter E (for "evenness"). Certain techniques, however, will be suggested that yield approximate values, and the coefficients so found will be designated E_r, E_t, E_{re}, and E_{te}. No two coefficients of these types are comparable unless comparable systems of areal distributors are used in finding them.

First, however, a few observations may be made for the benefit of any readers who may be tempted to try their hands at the problem of finding a method that will yield an exact value for E.

Since the degree of absolute areal evenness is a characteristic of a distribution that is independent of any particular system of areal distributors into which the distributary region happens to be divided, it cannot be found with the aid of areal distributors. Furthermore, it cannot be found unless the position of each individual distributee may be definitely located on a map of the region. If every distributee may thus be located, the assumption could then be made that each distributee was "displaced" from the position that it would occupy if the distributees were distributed with perfect areal evenness over the surface of the region, and the degree of *unevenness* of the distribution would depend on the distance and direction that every distributee has been so "displaced." The problem consists in devising a method of finding two measures of the amount of this hypothetical "displacement," M and Q, M showing the actual amount of displacement and Q the amount of displacement that would obtain with maximum unevenness of distribution. The technique of finding M would have to be developed in such a way that different degrees of absolute evenness would always be represented by correspondingly different values of M. This means that the distance and direction of "displacement" of every individual distributee would in some manner have to be correlated in M with the distance and direction of the "displacement" of every other distributee and also with the boundaries of the distributary region. Q, representing the maximum possible amount of "displacement," would always be a larger quantity than M and the coefficient of absolute areal evenness, E, would be $1-M/Q$.

For linear distributions, or distributions measured in terms of one dimension only, the technique described in connection with Formula 7 on page 193, accomplishes this and yields a definitive coefficient of absolute linear evenness (if the location of every distributee along the line is known). It seems reasonable to suppose that a similar technique could be adapted to the analysis of the evenness of areal distributions, or distributions measured in terms of two dimensions, but the problem of how so to adapt it as yet has defied solution.

E_r: If comparable values for distributions in different distributaries are not required, the mean (or the product) of two or more coefficients R_a based on different irregular systems of distributors of different scales, both coarse and fine, could be taken as a coefficient E_r. A coefficient E_r for the United States, for example, might be the mean (or the product) of coefficients R_a

determined with respect to the distribution of the population among the areas of counties, States, and larger geographical divisions respectively.

E_t: When comparable values for distributions in different regions are desired, the mean (or the product) of two factors could be taken in every case, the first factor being a simple coefficient T_a determined with reference to a system of distributors of fine scale (say 1/100) and the second a compound coefficient $\overline{T}_a$ found as suggested in the preceding section.

E_{re} and E_{te}: Coefficients E_r and E_t both fail to take into full account one characteristic of an areal distribution that must be considered if a more refined measure of the degree of absolute areal evenness is desired. This characteristic is the eccentricity of the distribution in relation to the center of the distributary region. If the coefficient of fine scale R_a or T_a that forms one of the factors in either E_r or E_t has a high value, the center of the distribution must be close to the center of the distributary. If, on the other hand, the fine-scale coefficient has a low value, the two centers might be near together or far apart. It is clear that, other things being equal, a distribution whose center lies near the center of the distributary is more even from the absolute point of view than is one whose center lies near the outer edge of the distributary.

The degree of eccentricity may be indicated by a factor f, whose value is 1 when the two centers coincide and 0 when they are as far apart as possible, or, in other words, when the center of the distribution lies on the outer edge of the distributary at a "point of maximum unevenness," i.e., at the maximum possible distance from the center of the distributary.

If, then, we wish to modify E_r or E_t by taking factor f into account, this might be done either by (1) multiplying these coefficients by f, (2) taking the mean of E_r or E_t and f, or (3) in the case of E_t by taking the mean of T_a, $\overline{T}_a$, and f (i.e., $\frac{1}{3}(T_a+\overline{T}_a+f)$). The coefficients thus derived may be designated E_{re} (a modification of E_r) and E_{te} (a modification of E_t).

Factor f is $1-W/D$, W being the distance separating the two centers and D the distance from the center of the distributary to the "point of maximum areal unevenness." The centers of population and of area may be found as follows:

(1) Superimpose a grid of rectangular coördinates on the base map and find the population, p, and the area, a, of every column, or vertical strip of territory, marked out by the lines within the region.

(2) Number the columns serially 1, 2, 3, etc., from the left side of the region to the right, and multiply the population and the area of each column by its serial number. Let Q be the sum of the products with respect to the populations and S be the sum of the products with respect to the areas.

(3) $Q/P - ½$ and $S/N - ½$ are the distances (in terms of the widths of columns) from the left side of the first column to the vertical ordinates that pass through the center of population and the center of areas, respectively, P being the population and N being the area of the region.

(4) The horizontal ordinates passing through the two centers are found in a similar manner, with respect to the rows marked out by the coördinates.

In finding these coefficients of the E type we have suggested that either the mean or the product of certain factors be taken. Which procedure is selected will depend upon the relative weight that we wish to give the several factors. When the products are taken the final value will be small if any one of the factors is small. If the means are taken the value of the final coefficient is much less affected by a low value on the part of any one of the factors.

The Association of Areal Distributions

The problems that we encounter in assessing the degree of association of two areal distributions are similar to those just discussed in connection with the evenness of areal distributions and similar methods of finding the coefficients may be employed.

As in the case of areal evenness a distinction may be made between the degree of absolute areal association and the degree of association relative to the system of areal subdivisions employed in the analysis.

When two elements, A and B, are evenly distributed over the same areas and neither element covers any territory not covered by the other there is perfect areal association between them. If either element is moved, the degree of association is reduced. If the position of one of the elements is shifted so much that the two elements cover wholly different areas there is, at least from one point of view, a complete absence of areal association or total areal disassociation.

This concept of minimum areal association cannot be applied to the distributions of individual objects such as human beings. Two human beings cannot occupy the same area; consequently when the distributions of two populations, such as whites and negroes, are conceived in terms of their smallest indivisible units, persons, the two elements always cover wholly different areas and there would always be a complete absence of areal association—which, of course, is absurd. The degree of absolute areal association, therefore, must be measured in terms of the degree to which two distributions approach perfect areal association rather than in terms of the degree of coincidence. Conversely, the degree of absolute areal disassociation would be the degree to which the two elements depart from perfect

association; but to carry this concept to its logical conclusion would also lead to absurdity, for the ultimate extreme of disassociation would, in this case, be reached only if one element were removed to an infinite distance from the other.

Hence if we wish to measure the degree of association a limit must be set to the degree of possible disassociation. This limit is established by the boundaries of the region within which the two distributions lie. In other words, the degree of absolute association of two distributions must always be expressed with reference to a region that includes the whole of both distributions. Absolute disassociation within any given region may then be defined as the relationship that would exist if the whole of elements A and B respectively were concentrated at two points on the outer edge of the region at the maximum possible distance from one another, and a coefficient of absolute areal association for any two distributions within the region would show the degree to which their association approaches perfect association when minimum association (or absolute disassociation as just defined) is taken as the point of departure.

Maximum *relative* areal association, shown by coefficient R_{as}, occurs when the ratio of the number of units of A (distributees) to the number of units of B (secondary distributors) is the same with respect to every distributor (or, in other words, is the same in every areal subdivision that contains any part of the distributary (element B)). Areal subdivisions that contain no part of the distributary (element B) exert no effect on the coefficient R_{as}. If both elements are wholly included in only one areal subdivision there is maximum relative association; there is no relative areal association and the coefficient is 0, if both elements are not represented together in at least one areal subdivision. As in the case of the coefficients of areal evenness, however, the value of any coefficient R_{as} may be influenced considerably by the manner in which the system of areal subdivisions employed breaks up the two elements. If a coarse-scale system is employed, the coefficient will show more in regard to the degree of association with respect to large parts of the region; a coefficient of fine-scale will disclose more concerning the degree of intimate or local association but may fail to take into account the larger areal relationships. If comparable coefficients of association are desired, almost precisely the same steps must be taken to obtain them as in the case of the coefficients of areal evenness, and comparable systems of areal subdivisions must be employed in the analyses. If we wish to compare the degree of areal association of two elements in one region with the degree of areal association of two elements in another region, the analysis in each region must be based upon a regular system of areal

subdivisions and the scales of both systems must be the same. (A coefficient of areal association based on a regular system of areal subdivisions may be designated $R_{as(t)}$ and one based on an irregular system $R_{as(r)}$). If a coefficient showing the degree of association with respect to large parts of a region is desired, the values must be determined for several different coarse-scale systems of areal subdivisions and the mean of these values taken ($\overline{R}_{as(t)}$ or $\overline{R}_{as(r)}$).

Coefficients similar to the evenness coefficients of the *E*-type that give approximate indications of the degree of absolute areal association may be found in much the same way as are the *E*-coefficients. A fine-scale simple coefficient $R_{as(r)}$ (or $R_{as(t)}$) and a coarse-scale compound coefficient $\overline{R}_{as(r)}$ (or $\overline{R}_{as(t)}$), may be determined and the means of these taken for a value As_r (or As_t) corresponding to E_r (or E_t). Two coefficients As_r for different regions are never comparable, but two coefficients As_t for different regions are comparable if based on comparable systems of areal subdivisions.

Furthermore As_r and As_t may be modified to yield values As_{re} and As_{te} corresponding to E_{re} and E_{te} respectively, in which the element of eccentricity is considered. This is done by introducing a factor of eccentricity, f_{as}, as in the case of the evenness coefficients; f_{as} is $1 - G/H$, G being the distance between the centers of the two distributions A and B and H being the distance between the two points on the outer edge of the region that are farthest apart from one another—or, in other words, the distance that would separate the two centers with minimum possible association of the two distributions within the region.

Summary of Areal Coefficients

The properties of the several areal coefficients may be summarized briefly.

(1) The coefficients based on irregular systems of areal subdivisions (i.e., R_a, E_r, E_{re}, $R_{as(r)}$, As_r, and As_{re}) may be used only in comparing the degree of evenness or of association of different distributions within a given region and then only when all the coefficients to be compared are based on the same system or systems of areal subdivisions. Such coefficients, however, are easier to compute than the coefficients based on regular systems of areal subdivisions (i.e., T_a, $\overline{T}_a$, E_t, E_{te}, $R_{as(t)}$, As_t, As_{te}) since it is not

[21] The product should not be taken in this case since the fine-scale factor $R_{as(r)}$ or $R_{as(t)}$ could conceivably be 0, which would reduce the value of As_r or As_t to 0 regardless of the value of the coarse-scale factor. In the case of evenness coefficients T_a never has a value of 0.

necessary to subdivide the regions into equal areal subdivisions and to reapportion the statistical data accordingly. If, however, comparable coefficients for different regions are required, they must always be based on comparable regular systems of areal subdivisions for all the regions concerned.

(2) If one wishes to determine how evenly the population of a given region is distributed:

(a) among relatively small subdivisions of the region, regardless of how the several subdivisions of differing densities of population are themselves distributed, a simple coefficient T_a or R_a of fine scale is employed;

(b) among relatively large subdivisions of the region, regardless of how the population is distributed within each of the subdivisions, a coefficient T_a or R_a (or compound coefficient $\overline{T}_a$ or $\overline{R}_a$) of coarse scale is employed;

(c) among *both* large and small subdivisions of the region, regardless of whether or not the population as a whole tends to be centrally located in the region, a coefficient E_r or E_t is employed;

(d) among both large and small subdivisions of the region, and if one also wishes to take into account the degree to which the population is centrally located in the region, a coefficient E_{re} or E_{te} is employed.

(3) If one wishes to determine the degree of areal association of the distributions of two elements A and B:

(a) within relatively small subdivisions of a region, regardless of whether there is a high or low degree of association within larger subdivisions of the region, a simple coefficient $R_{as(t)}$ or $R_{as(r)}$ of fine scale is employed;

(b) within relatively large subdivisions of the region, regardless of the degree of areal association within smaller subdivisions of the region, a coefficient $R_{as(t)}$ or $R_{as(r)}$ (or compound coefficient $\overline{R}_{as(t)}$ or $\overline{R}_{as(r)}$) of coarse scale is employed;

(c) within both large and small subdivisions of the region, regardless of the relative positions of the centers of the two distributions, a coefficient As_r or As_t is employed;

(d) within both large and small subdivisions of the region, and if one also wishes to take into account the relative positions of the two centers, a coefficient As_{re} or As_{te} is employed.

CONCLUSION

The various techniques and coefficients discussed in this paper are merely tools for measuring relationships. Some take more factors of relationship into account than others. In this discussion we have been concerned with

the tools themselves rather than with what can or cannot be done with them. Obviously in studying any particular geographical problem the right tool should be used. Unless the relationships disclosed by a coefficient have meaning and importance the coefficient may be misleading. For example, a coefficient E_{te}, which shows (approximately) what we have called the degree of absolute areal evenness, would be inappropriate in measuring the degree of evenness of the distribution of population over the area covered by one of the sheets of a topographic map. Coefficient E_{te} takes account not only of the way in which the population is distributed among subdivisions of a region but also of the positions of these subdivisions in relation to the boundaries of the region. In the case of a region bounded by arbitrary lines the relationship of a distribution to these lines is of no significance whatever. Whether a city happens to be near the edge or near the center of a topographic quadrangle means nothing, and a coefficient of evenness that is influenced by such an accidental relationship should not be employed. If, therefore, we wish to measure and compare the evenness of distributions in different sample areas bounded by arbitrary lines, we should employ coefficients T_a, or $\overline{T}_a$, or E_t, depending on the characteristics of the distributions that we wish to study. There are, however, cases where the region with respect to which a distribution is analyzed has a definite functional relationship to the distribution itself. The range of a biological species (i.e., the geographical territory that it occupies) has a functional relationship to the distribution of the species. The territory served by a wholesale grocery establishment has a functional relationship to the distribution of the retail stores that purchase from the wholesale house. The territory of a nation has a functional relationship to the distribution of the nation's population. In cases of this sort it might be appropriate to employ a coefficient that takes into account the degree to which the distribution tends toward concentration near the center or near the margin of the region (i.e., a coefficient E_{te} or E_{re}).

In short, the question of how the different coefficients may best be used opens up interesting avenues for further investigation. It is hoped that the material set forth in this paper may suggest to readers possible ways of refining and improving the techniques and also of applying them.

American Geographical Society.
September, 1937.

On the Treatment of Surface Features in Regional Studies

PRESTON E. JAMES

The distinguishing feature of an adequate technique in modern geographical studies is the attempt to establish as accurately as possible on maps the distribution and space relations of the material facts of the landscape with which the study is preoccupied. Possibly no facts are more fundamental in such studies than the facts of surface configuration; yet a review of recent literature suggests that this element receives, in general, anything but adequate treatment. It seems desirable, therefore, to take stock of the methods commonly in use in regional or areal studies, and to summarize the suggestions which have, from time to time, been made regarding this important element. Our attention is focused on the treatment of surface features as a part of the description and interpretation of areas, not as a topical specialty.

HYPSOMETRIC MAPS AND SURFACE CONFIGURATION MAPS

The commonest method of representing the form of the land surface is through the use of the hypsometric or contour map. On a "topographic" scale (larger than 1/100,000)[1] the portrayal of surface form by the use of contours is probably as effective as anything yet invented. But if one wishes to go beyond surface form to the geognosy or soil, one finds that the critical changes in these elements are not indicated by the contours. Maps of the sort used by Schmieder in the study of Guadalupe Valley[2] or Wilson

[1] The writer uses the terms topographic, chorographic, and geographic in the sense of increasing orders of magnitude. A topographic map is one of a small area, on a scale of not less than 1/100,000. A chorographic map is one of intermediate scale. A geographic map is one of the world as a whole. See P. E. James, "The Terminology of Regional Description," *Annals of the Association of American Geographers*, Vol. 24 (1934): 78–86.

[2] O. Schmieder, "Lower Californian Studies. II. The Russian Colony of Guadalupe Valley," *Univ. Calif. Pub. in Geog.*, Vol. 2 (1928): 409–434. Map I of surface forms and geognosy.

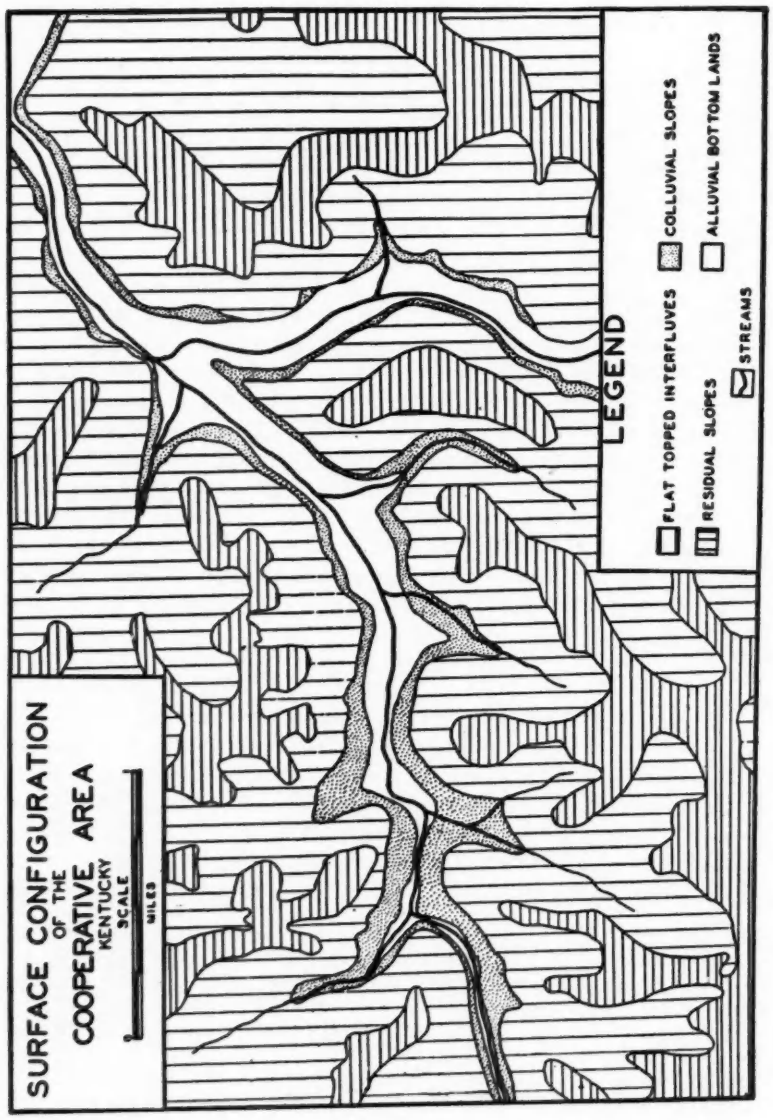

FIG. 1—Map of a part of the Cumberland Plateau in Kentucky (Leonard S. Wilson).

in a study of a portion of the Cumberland Plateau in Kentucky (Fig. 1)[3] make use of surface-soil types rather than of contours alone.

On a chorographic scale, however, the use of lines of equal altitude to reveal surface form is not at all adequate. Hypsometric maps, whether colored or made in black and white, are notoriously difficult to read if one wishes to gain an exact picture of surface form. The example frequently given to illustrate this point is taken from the map of that portion of the United States which includes the front of the Rocky Mountains and the western part of the Great Plains. Denver, for instance, occupies its position a mile above sea level, but nevertheless incontestably outside of the mountain country. On a map of altitudes the shading which indicates higher elevation (and so, generally, mountainous country) includes not only Denver but a considerable portion of the plains east of Denver. No one isohypse marks the edge of the mountains, yet the most important feature to set forth on a map of surface configuration is not some arbitrary line of equal altitude, but rather the line along which the plains are bordered by the Rockies. Such lines are not clearly and unmistakably revealed on hypsometric maps on a chorographic scale because the vertical interval between contours is necessarily too great.

EXAMPLES FROM RECENT GEOGRAPHICAL WRITINGS

The best way to approach this argument is to review the treatment of surface features in recent geographical writings. If the reader will turn to the articles mentioned and examine the maps he will see more clearly than can be presented in words alone the relative effectiveness of the treatment of surface features. The examples have been picked at random from recent geographical publications in America: they do not at all represent the results of an exhaustive survey. If the German and French literature had been similarly treated it would reveal about the same range from effective to ineffective.

L. Dudley Stamp's treatment of Burma[4] is a good example of regional description. The two maps on which the surface features are shown, one of altitudes, the other of natural regions, are clearly drawn, with considerable detail (Figs. 2 and 3). They offer, therefore, the best possible illus-

[3] L. S. Wilson, "The Geography of a Representative Portion of the Northwest Cumberland Plateau," manuscript included in the *Papers of the Mich. Acad. of Sci., Arts and Letters,* 1937 (To be published in 1938).

[4] L. Dudley Stamp, "Burma, An Undeveloped Monsoon Country," *Geogr. Rev.*, Vol. 20 (1930): 86–109. The two maps to which reference is made are on pages 88 and 89. They are reprinted herewith as Figures 2 and 3.

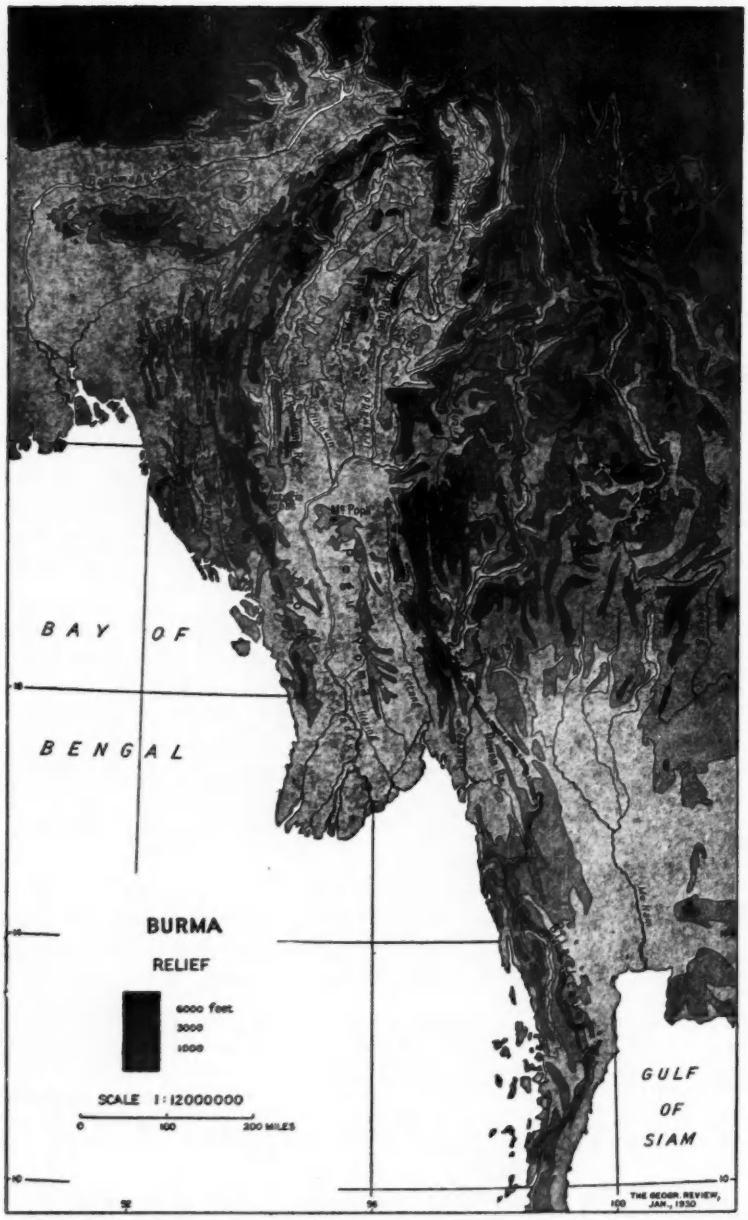

FIG. 2—Burma—hypsometric map (L. Dudley Stamp). Courtesy of the *Geographical Review* published by the American Geographical Society of New York.

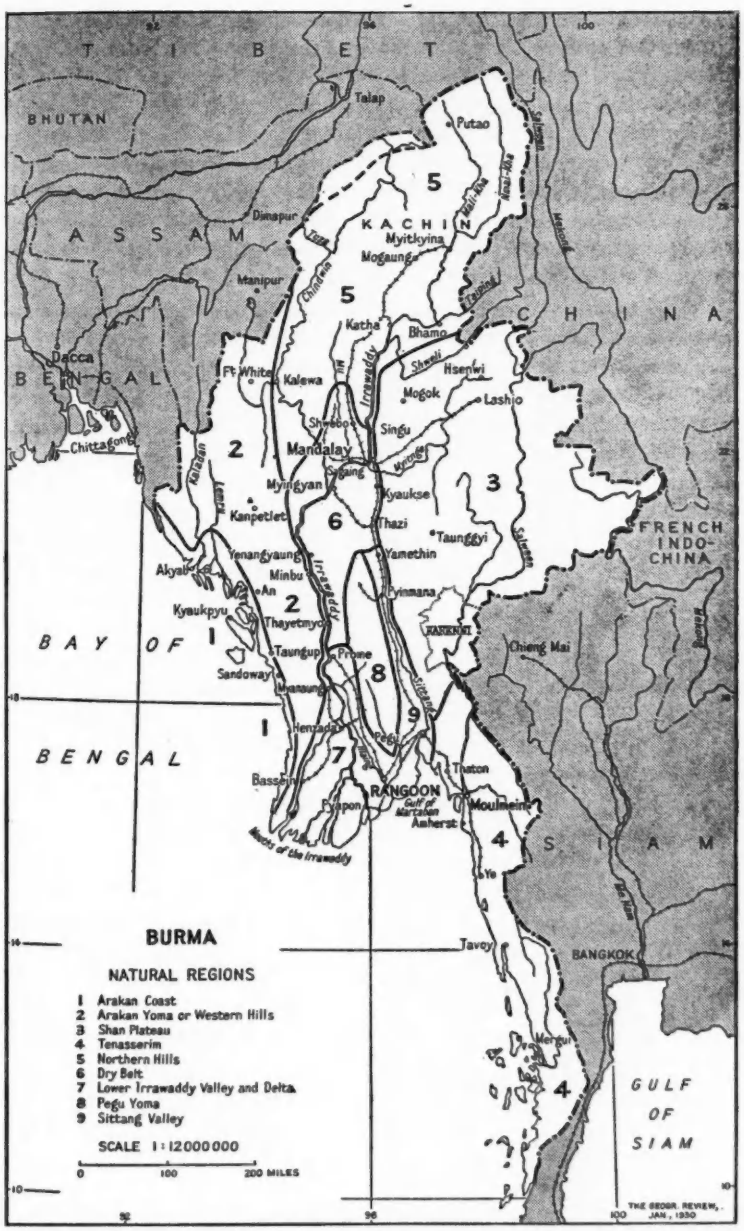

FIG. 3—Burma—natural regions (L. Dudley Stamp). Courtesy of the *Geographical Review* published by the American Geographical Society of New York.

tration of the argument herein set forth—namely, that hypsometric maps, even the best of them, are not effective in the portrayal of surface form on a chorographic scale. If the reader will examine these maps closely he will see, for example, how impossible it is to tell from the altitude map alone how far south the Arakan Yoma extends. Altitudes above 1,000 feet are not found much south of the latitude at which the Irrawaddy River starts to bifurcate: but the natural region map on the opposite page shows this belt of hills extending far south of this. In so far as the natural region map is a map of surface configuration it shows much more effectively than the hypsometric map the critical boundaries of the surface features. Unfortunately the natural region map attempts to combine surface and vegetation character, with the result that the boundaries of both are obscured. For instance from neither the hypsometric nor the regional map can one tell exactly where the northern limit of the Pegu Yoma comes, since the northern part, which includes Mt. Popa, is cut off by a vegetation division. A simple surface configuration map, perhaps combined with vegetation symbols, would have brought out these critical distributions much more clearly.

Van Valkenburg uses a hypsometric map in his study of Java,[5] from which no clear idea of the exact areas of mountains and plains can be read. The plains are much better shown on the map of the geology in the same paper (p. 569).

Zierer, in a recent study of San Fernando,[6] uses a hypsometric map to show the surface features (p. 2). From this map it is quite impossible to fix the exact boundaries between mountain, alluvial fan, and basin floor; as the reader can easily prove by attempting this first on the hypsometric map, and then turning to another map (p. 4) on which the critical boundaries rather than arbitrary elevations are shown.

Torbert presents a small inset map at the beginning of his paper on the Santa Clara Valley,[7] on which the altitudes above 1,000 feet are shaded, leaving blank what one may suppose to be the valley floor. That this impression of the outline of the valley floor is much in error can be seen from other maps in this same paper, or by studying Broek's map[8] of the same area

[5] S. Van Valkenburg, "Java . . .," *Geogr. Rev.*, Vol. 15 (1925): 563–583. Relief map on p. 564; map of geology on p. 569.

[6] C. M. Zierer, "San Fernando—A Type of Southern California Town," *Annals of the Assoc. of Amer. Geogs.*, Vol. 24 (1934): 1–28. Relief map on p. 2; surface features shown on map p. 4.

[7] E. N. Torbert, "The Specialized Commercial Agriculture of the Northern Santa Clara Valley," *Geogr. Rev.*, Vol. 26 (1936): 247–263. Map on p. 247.

[8] J. O. M. Broek, *The Santa Clara Valley, California*, Utrecht, 1932. Map of surface features, Plate I.

on which hachures over the contours are used to show the extent of the mountain slopes.

In McBride's book on Chile,[9] a map of altitudes is used to show the surface divisions of the country. This map is reproduced here as Figure 4. Another map (Fig. 5), recently drawn by the writer, is presented to illustrate the impossibility of reading accurately from the hypsometric map alone the exact outline of so critical a feature as the Central Valley.

R. S. Platt, in his numerous studies of the details of occupance pattern, varies in his treatment of the underlying surface form. His paper on the Maracaibo Basin[10] includes two maps indicating the arrangement of the basin and surrounding highlands—but the boundary between highlands and basin, perhaps based on some generalized altitude limit, fails to give a true picture of the shape of this area. In his paper on the Brazilian coffee plantations[11] he attempts to portray the surface features of São Paulo state with a hypsometric map, which does not suggest even in vague outline the critical pattern of surface features. On the other hand, Platt makes excellent use of surface configuration maps in his treatment of Lima and vicinity[12] and includes a critical surface boundary in his maps of Magdalena Atlipac.[13]

More recently Pierre Deffontaines in his paper on mountain settlement in southeastern Brazil attempts to indicate the distribution of mountains by stippling the area above 1,000 meters.[14] Interesting and valuable as this paper is, its failure to show the arrangement of the mountains exactly on a map leaves the contribution seriously incomplete; for if there is any one criterion by which one may identify an example of "modern" geographical writing it is the attempt to establish and to set forth on a map as accurately as possible the distribution of the things, natural or human, which are being discussed.

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[9] G. M. McBride, *Chile, Land and Society,* Am. Geogr. Soc., Research Series no. 19, New York, 1936. Map, p. 25.

[10] R. S. Platt, "Pattern of Occupance in the Maracaibo Basin," *Annals of the Assoc. of Amer. Geogs.,* Vol. 24 (1934): 157–173. Maps on p. 157 and p. 164.

[11] *Idem,* "Coffee Plantations in Brazil," *Geogr. Rev.,* Vol. 25 (1935): 231–239. General map of the state on p. 232, and of individual coffee fazendas, for example, on p. 233. Compare the writer's map of surface configuration of this same area in P. E. James, "The Coffee Lands of Southeastern Brazil," *Geogr. Rev.,* Vol. 22 (1932): 225–244; map on p. 232; compare also the map of an individual fazenda, p. 237.

[12] *Idem,* "Six Farms in the Central Andes," *Geogr. Rev.,* Vol. 22 (1932): 245–259. Map on p. 255.

[13] *Idem,* "Magdalena Atlipac—A Study in Terrene Occupancy in Mexico," *Geogr. Soc. of Chicago,* Bull. no. 9, Chicago, 1933.

[14] P. Deffontaines, "Mountain Settlement in the Central Brazilian Plateau," *Geogr. Rev.,* Vol. 27 (1937): 394–413; maps on pp. 396 and 397.

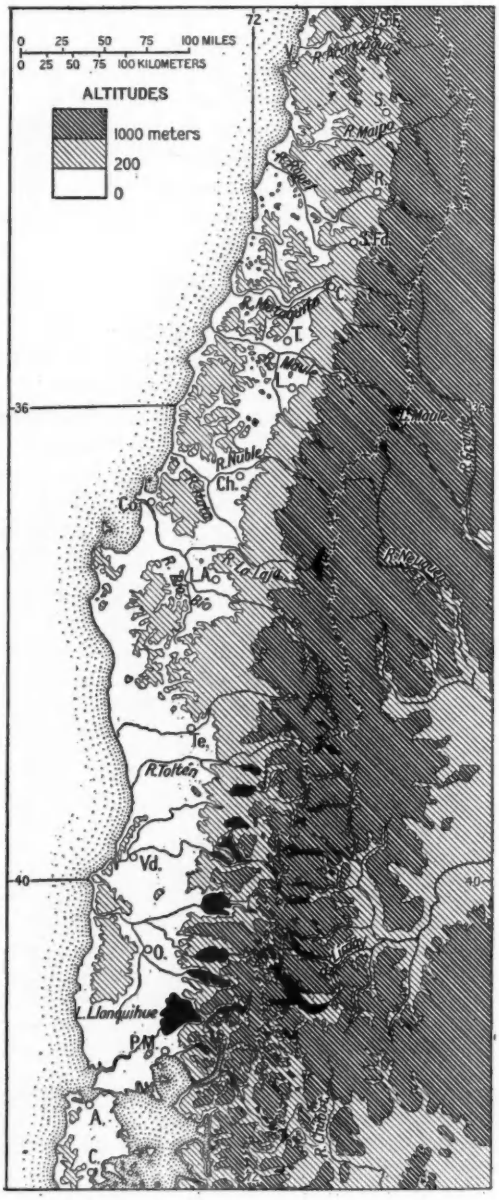

FIG. 4 — Hypsometric map of Middle Chile (George W. McBride). Courtesy of the American Geographical Society of New York, *Special Publications.*

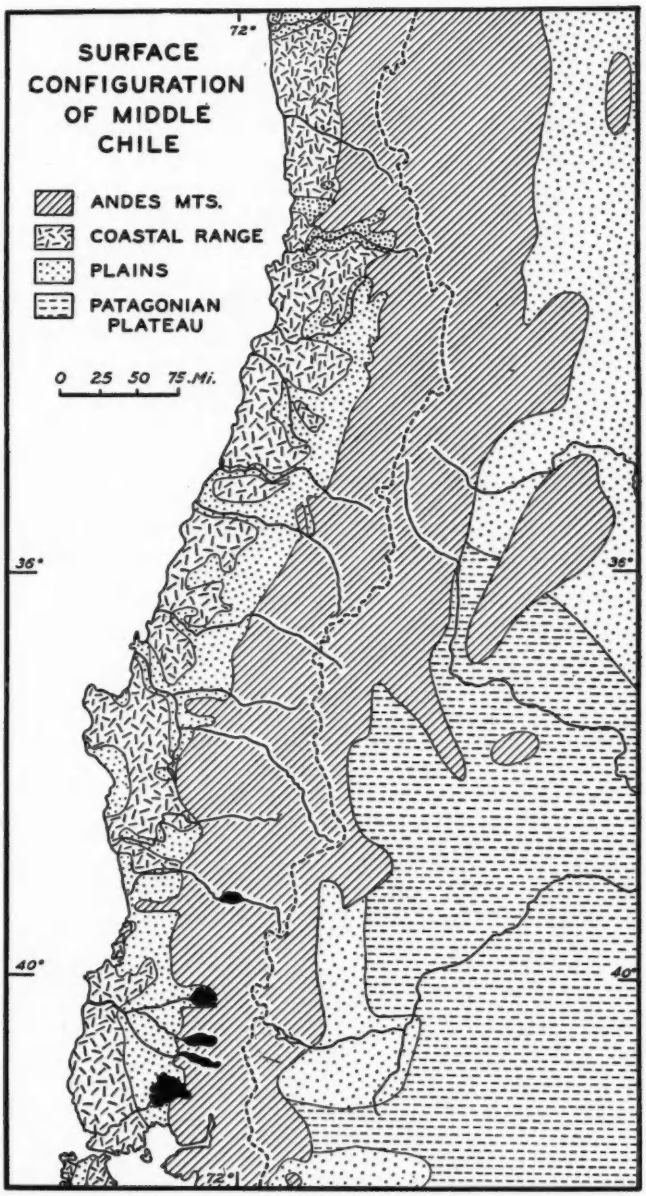

FIG. 5—Surface configuration of Middle Chile (Preston E. James).

A number of papers may be suggested at random as examples of the effective portrayal of surface features. The map of surface configuration in Hall's Yamato Basin[15] is a first-class example. The division of Japan as a whole into mountains, diluvial terraces, and alluvial lowlands by Trewartha presents clearly the essential elements in that complex terrain.[16] Dicken separates mountains and basins clearly in a paper on the Sierra Madre Oriental.[17] The boundary between mountains and plains can be clearly read from maps in Hartshorne's study of the boundaries of Upper Silesia.[18] D. H. Davis' maps of occupance patterns in Hokkaido[19] are much more effective than would be the case if the contours were omitted. In the latter case, the large scale of the maps permits the use of contours of relatively small vertical interval. How far such maps may be effectively reduced is suggested by Trewartha's study of Suwa Basin.[20] Zimmerman's paper on Siam includes three maps on the same page showing "morphology," vegetation, and population—maps which are easily comparable, and which therefore present the essentials of the Siamese chorography very effectively. The map of "morphology" is of surface configuration.[21]

QUALITATIVE METHODS OF TREATING SURFACE CONFIGURATION

A distinction must be made between qualitative treatments of surface features and treatments which are quantitative. Obviously the latter are much to be preferred, since only where the boundaries between surface categories are lines of equal numerical value can they be fitted into a thoroughly objective descriptive scheme. The quantitative methods, however, depend on the existence of accurate detailed hypsometric surveys, and in many of the regions being studied by geographers such surveys are not available.

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[15] R. B. Hall, "The Yamato Basin, Japan," *Annals of the Assoc. of Amer. Geogs.*, Vol. 22 (1932): 243–290. Map p. 248.

[16] Glenn T. Trewartha, *A Reconnaissance Geography of Japan*, Univ. of Wisconsin Studies in the Social Sciences and History, no. 22, Madison, 1934. Map p. 10.

[17] S. N. Dicken, "The Basin Settlements of the Middle Sierra Madre Oriental, Mexico," *Annals of the Assoc. of Amer. Geogs.*, Vol. 26 (1936): 157–178. Map p. 161.

[18] R. Hartshorne, "Geographic and Political Boundaries in Upper Silesia," *Annals of the Assoc. of Amer. Geogs.*, Vol. 23 (1933): 195–228. Map p. 200.

[19] D. H. Davis, "Type Occupance Patterns in Hokkaido," *Annals of the Assoc. of Amer. Geogs.*, Vol. 24 (1934): 201–223. The map on p. 201 could have included surface configuration with good effect. Reference is to numerous smaller maps.

[20] Glenn T. Trewartha, "The Suwa Basin: A Specialized Sericulture District in the Japanese Alps," *Geogr. Rev.*, Vol. 20 (1930): 224–244.

[21] Carle C. Zimmerman, "Some Phases of Land Utilization in Siam," *Geogr. Rev.*, Vol. 27 (1937): 378–393. Maps on p. 382.

The problem arises, then, concerning the categories of surface features which can be identified in a qualitative treatment, where adequate detailed surveys are either entirely lacking or cover too small a proportion of territory to be of much value.

On a scale which remains within the limits which the writer describes as "topographic" (a scale larger than 1/100,000), the details which can be recognized and mapped are greater than in studies of larger areas on smaller scales. The landforms of a territory strongly marked by river cutting can usually be resolved into: (1) valley bottoms; (2) intermediate slopes; and (3) interfluves.[22] In glacial country, on the other hand, these categories may be obscured by strongly developed features of glacial deposition—moraines of various surface expression (such as "knob and basin" or "sag and swell"), outwash plains, or numerous smaller forms commonly recognized by physiographers. In karst areas the physiographers have provided a terminology for sinks of different surface form, and for other features commonly encountered in such terrain. In regions where the underlying geologic structure results in prominent surface features (for example, cuestas, mesas, or fault line scarps) these features can be recognized and mapped.

On a somewhat smaller scale of mapping, covering larger extents of territory, many of these details must be generalized into larger units. Here the physiographers have led the way with their "physiographic regions." Fenneman's regions of the United States[23] form the standard source for the maps of surface configuration since they are based largely on contrasts in surface form. Atwood's "natural regions"[24] are of a similar sort, although generalized for pedagogical purposes. It is not uncommon, however, for persons who are drawing physiographic regions to base their boundaries on changes in the geologic structure, whether, or not, these changes are, in every instance, coincident with surface changes. In detail, this difference may be illustrated by the western boundary of the Cumberland Plateau in Kentucky and Tennessee. Whether the boundary is to be placed at the western limit of the sandstones, or at the base of the Pottsville Escarpment may involve, in some places, practically no difference, but in other places

[22] These and other categories were used in the writer's study of a section of New England: P. E. James, "The Blackstone Valley . . .," *Annals of the Assoc. of Amer. Geogs.*, Vol. 19 (1929): 67–109. See also H. M. Kendall, "The Occupance of the Lower Vézère Valley," *Papers of the Mich. Acad. of Sci., Arts and Letters*, Vol. 26 (1931—published 1932): 299–312. Maps on pp. 306 and 310.

[23] N. M. Fenneman, "Physiographic Divisions of the United States," *Annals of the Assoc. of Amer. Geogs.*, Vol. 18 (1928): 261–353.

[24] W. W. Atwood, Series of continental wall maps, Chicago.

two lines thus drawn would depart significantly from each other. A surface configuration map would recognize surface difference rather than geologic difference if the choice involved any important distinction.

Certain geographers, perhaps more gifted in the use of the pen, have made striking use of relief-like symbols on their maps of surface configuration. Outstanding in this group are Lobeck,[25] Guy-Harold Smith,[26] and Raisz.[27] This method may combine pictorial quality with accuracy of division between categories.

The definition of surface categories can be approached also from the smallest scale maps, recognizing the broadest generalizations of the face of the earth. At such scales the major features of the earth's surface are the continents and ocean basins, the earth's major lineaments. The surface of each continent, however, is marked off in a characteristic and unique pattern of surface configuration. These surface features are so much a part of the layman's experience that words for the chief categories are already at hand in the popular vocabulary. Plains, plateaus, hills, and mountains are basic concepts in spite of the rather vague definitions given them in popular usage. If they are to be used in a technical sense in geographic description some attempt must be made to sharpen the definitions. The following statements are suggested for this purpose:

A *plain* is an area of relatively slight local relief (less than 300 feet within an area of approximately 25 square miles): it is low-lying with reference to surrounding areas, and is usually, but not in every case, low in altitude.

A *plateau* stands distinctly above neighboring areas, at least on one side, and has a considerable part of its surface at or near the summit level. Its local relief may or may not exceed 300 feet. A plateau, from the point of view of its physiographic background, may represent either a youthful land, recently uplifted and with a considerable portion of its initial surface still preserved, or it may represent an area protected from more rapid denudation by horizontal or nearly horizontal layers of stratified rock. It may, also, be surmounted by mountain ranges; but narrowly inclosed surfaces of slight local relief and high altitude, which are in many cases called plateaus, probably should be designated as mountain basins.

A *hilly upland* differs from a plateau in that a considerable part of its surface does not remain near the summit level. Its profiles, therefore, are rounded or crested rather than flat-topped. Uniformity of summit level, as in the case of many dissected plateaus, is a not uncommon characteristic of this type of surface. Many

[25] A. K. Lobeck, *Physiographic Diagram of the United States*, Chicago, 1921.

[26] Guy-Harold Smith, *Physiographic Diagram of South America*, New York, 1935; *idem*, *"Physiographic Diagram of Japan," Geogr. Rev.*, Vol. 24 (1934): Plate II.

[27] E. J. Raisz, "The Physiographic Method of Representing Scenery on Maps," *Geogr. Rev.*, Vol. 21 (1931): 297–304.

of the "crystalline oldlands" belong in this category. The hilly upland differs from a plain in degree of local relief, having more than 300 feet and less than 1,000 feet.

A *low mountain*, it seems, should be distinguished from a high mountain. This feature is similar to a hilly upland in profile, but has a local relief of more than 1,000 feet and generally less than 3,000 feet.

A *high mountain* is similar to the last two categories in that it possesses a rounded or peaked summit (as opposed to flat-topped plateaus—such as Mt. Roraima in Guiana, which should be classified as a plateau). A satisfactory working definition of high mountains in the low and middle latitudes bases the recognition of this feature on the existence of a marked vertical differentiation of the vegetation cover.[28]

In a number of instances these major divisions might well be subdivided. Because of detailed differences in the landforms crystalline areas should, generally but not always, be distinguished from areas with stratified rocks. Glaciated terrain is markedly different from non-glaciated terrain. In the low and high mountain areas the intermont basins should be separated from the rougher surfaces. As the area to be treated becomes smaller, the number of such more detailed subdivisions becomes greater.

In any system of generalization the question of boundaries is apt to become troublesome. For example, should the foothills of a high mountain system—the marginal cuesta belt, let us say—be included with the mountains or indicated as a separate surface division? For two reasons the former procedure seems preferable in smaller scale chorographic studies. In the first place, the data on which to separate the marginal areas are not in every case available. But even if they are, the question of logic might be raised at an attempt to treat the boundary of a generalization too specifically. Foothills, therefore, probably should be included with the mountains, unless they are essentially different; the dissected (and perhaps no longer flat-topped) edges of plateaus should be included with the plateaus; or the low "rolling" surface on the margin of a hilly upland, even if it has a local relief of less than 300 feet, should be included with the upland. In drawing the boundaries the attempt is made to identify the critical changes in surface character, and here to establish the dividing lines rather than to apply any arbitrary quantitative definition based on altitude or relief alone.

Practically speaking, each individual area must be considered as a separate problem. The geographer must learn to select the critical features of the surface which exert an influence on the pattern of the landscape as a

[28] These statements are presented in P. E. James, *An Outline of Geography*, Boston, 1935, p. 66; the use of vegetation differentiation for the definition of high mountains is discussed on pp. 315–318. See also the similar definitions, with added detail, in V. C. Finch and Glenn T. Trewartha, *Elements of Geography*, New York, 1936; definitions in footnotes on pp. 321 and 443.

whole and must proceed to map these features as effectively and clearly as possible.[29]

QUANTITATIVE METHODS OF TREATING SURFACE CONFIGURATION

Persons who have attempted to treat the surface features of a region quantitatively have focused their attention on various devices for measuring the degree of slope. Among the first to depart from the use of contour maps was the Michigan Land Economic Survey[30] which produced, as one of the maps setting forth the geographic inventory of the counties it surveyed, a map entitled "Lay of the Land." This was a map of degree of slope. The *unit area* method of land inventory now incorporates this feature as a part of the areal complex.[31]

For the treatment of larger areas at a smaller scale a new European method set forth in America by Guy-Harold Smith is of great value.[32] On a map of Ohio on a scale of 1/600,000, Smith plotted rectangles of five minutes of latitude and five minutes of longitude, each being approximately 25 square miles in area. Referring then to the topographic maps of the U. S. Geological Survey (which completely cover that State) he noted the difference between the highest and lowest points in each rectangle. Plotting these figures for each rectangle on the State map, he then drew isarithms (isopleths) for each hundred feet of relative relief. The resulting map is reproduced here as Figure 6. Using this method the smaller the rectangles the more detailed are the results, but for each scale of map there is probably a lower limit beyond which smaller rectangles only obscure the essential facts by adding intricacy.

A recent paper by Raisz and Henry[33] offers a modification of Smith's

[29] For examples of the writer's treatment of surface features in regional papers see: "A Geographic Reconnaissance of Trinidad," *Econ. Geog.*, Vol. 3 (1927): 87–109, map on p. 87; "The Surface Configuration of Southeastern Brazil," *Annals of the Assoc. of Amer. Geogs.*, Vol. 23 (1933): 165–193, including several maps; "Regional Planning in the Jackson Hole Country," *Geogr. Rev.*, Vol. 26 (1936): 439–453, map on p. 440.

[30] Department of Conservation, series of county maps, chiefly of the northern part of the state, Lansing, Michigan.

[31] V. C. Finch, *Montfort: A Study in Landscape Types in Southwestern Wisconsin*, Geogr. Soc. of Chicago, Bull. 9, Chicago, 1933. See also the maps of the Geographic Section, Land Planning and Housing Division of the Tennessee Valley Authority.

[32] Guy-Harold Smith, "The Relative Relief of Ohio," *Geogr. Rev.*, Vol. 25 (1935): 272–284; includes a résumé of the European studies on this subject.

[33] Erwin Raisz and Joyce Henry, "An Average Slope Map of Southern New England," *Geogr. Rev.*, Vol. 27 (1937): 467–472. The writer does not agree at all with the limitations suggested by these authors on the use of Smith's method—see p. 468.

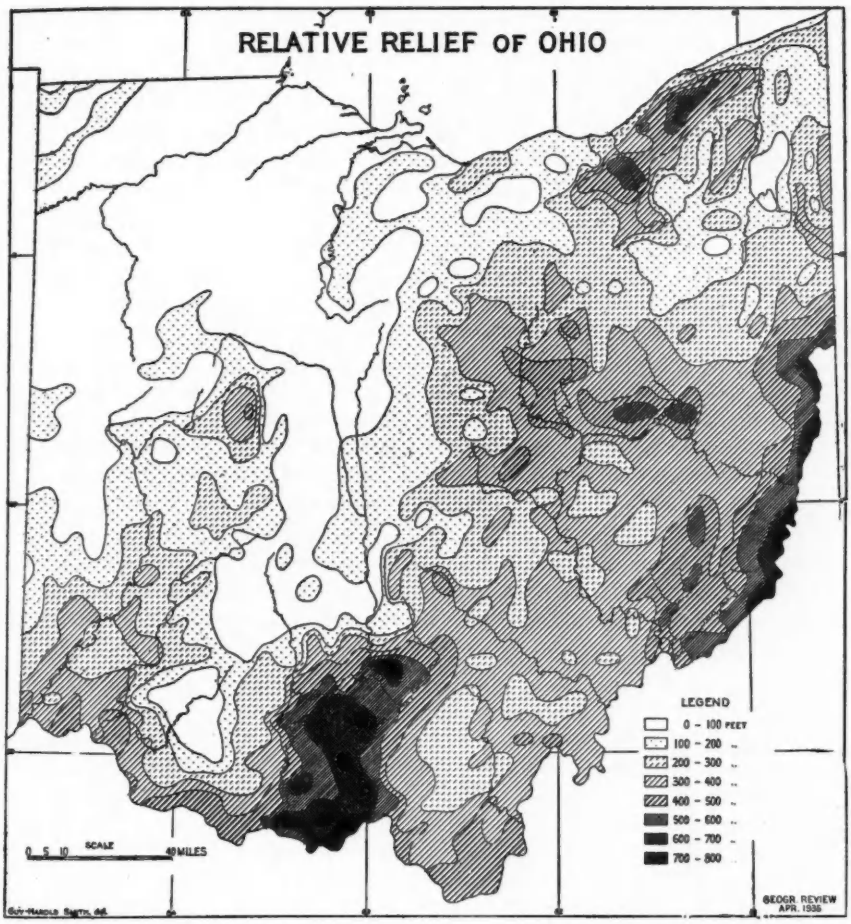

FIG. 6—Relative Relief of Ohio (Guy-Harold Smith). Courtesy of the *Geographical Review* published by the American Geographical Society of New York.

method. Southern New England is separated into areas of roughly the same slope as revealed by the density of the contours. The areas are of irregular size and shape, but are designed to leave in one unit such essential features as isolated monadnocks, intervales, or deeply incised valleys. Each area is then classified among six categories according to the average spacing of the contours. This method is not so strictly objective as the one pro-

posed by Smith, but it succeeds in emphasizing the pre-conceived surface divisions of the area (such divisions as would be identified in a qualitative treatment), and then of giving these divisions a quantitative expression. For the purpose of discovering the essential character of the land surface, Smith's more objective method would seem preferable: briefly, it is an excellent laboratory method. For the purpose of presenting the results, however, it is possible that Raisz' method would have the advantage of making the basic elements of the surface stand out more distinctly. Smith's map of relative relief would show these features more realistically but not so clearly —for in the actual landscapes the various parts of the terrain are by no means so sharply separated one from another as they are, and are quite properly, in a geographical description.

CONCLUSION

From this discussion it can be seen that geographers are by no means unaware of the problems involved in the cartographic representation of surface features, and that some experiment in new methods is from time to time attempted. The full fury of criticism must, therefore, be aimed at those who are content to portray surface form by hypsometric maps alone. This conventional procedure is demonstrably inadequate to provide that accuracy of detail, excepting at the largest scales, on which modern geographic studies must be based.

University of Michigan,
June, 1937.

Conditions of Exceptional Visibility within High Latitudes, Particularly as a Result of Superior Mirage

WILLIAM HERBERT HOBBS

The occasional very exceptional visibility within the polar regions, while in a general way familiar to most men with experience of the higher latitudes, is little known to others; and the magnitude and importance of these phenomena are but little appreciated even by those who have spent much of their lifetime within these less hospitable regions.

A rather general expansion of the range of vision within the polar areas is of course in part to be explained by the submergence of most polar lands under ice and snow, and the consequent small amount of dust or other minute solid particles which are suspended in the air. It is of great importance, also, that over and about the great land masses of Greenland and the Antarctic, the prevailing winds are outward and the air which rises from them is low in moisture content, which also favors visibility.

Wherever range of vision is not limited by earth rotundity, as when flying high in aircraft, objects at quite astounding distances have been clearly seen, as a few examples will suffice to show. Admiral Byrd when flying on the depot-laying trip in 1929, and again on his flight to the South Pole, clearly saw from over the Liv Glacier the mountains which lie to the west of the Beardmore Glacier at a distance of about 185 geographical miles, and his pictures showed objects as much as 200 miles away. Watkins, flying off the East Greenland Coast with Lieutenant D'Aeth at an elevation of about 4000 feet, saw Cape Dan at a distance of 200 miles, and so clearly that had it not pushed out so far, other headlands much more remote must have been seen. Sir Hubert Wilkins from his plane when flying in the polar regions has, he believes, several times seen objects distant as much as 200 miles. The Dutch flyer, Van Giesen, reports that in 1932, when he flew much over and about Iceland, he saw objects at a distance of 200 miles as clearly as objects are usually seen in Europe at distances of only 20 miles. Captain Riisar Larsen has supplied me with another instance from a flight made off the Antarctic coast.

If the observer is on or but little elevated above the surface of the earth, as when on a ship at sea, the earth's rotundity usually sets quite definite

bounds to the range distance of vision. At sea it is well recognized that the atolls with an elevation of 12 to 15 feet can be made out at distances of about twelve miles. The top of a mountain 3000 feet high is just eclipsed at a distance of 72 statute miles, one 4000 feet high at 83 statute miles, and one 5000 feet high at 93 statute miles. If objects are seen at greater distances it is only as a result of superior mirage, or "looming," which brings them up from below the horizon. This phenomenon occurs with certain atmospheric conditions only, and may come on quite suddenly or disappear as suddenly with modification of the atmospheric condition.

A good example of such raising of objects from below the horizon is supplied by Commander D. B. MacMillan, who on May 24, 1909, saw and clearly recognized Capes Joseph Henry and Hecla in Grant Land from Cape Washington on the North Greenland coast, though these headlands were at a distance of about two hundred miles. From Mount Evans, the meteorological station of the University of Michigan Expedition in Greenland, which had a limited horizon on most occasions of perhaps a dozen miles, it was possible to see on certain days complete panorama of glaciers far beyond the normal horizon to the southward.

This peculiarly characteristic polar, or *superior* mirage, is strongly contrasted in nature with the much better-known desert type, or *inferior* mirage. The latter is due to layers of hotter air which lie close to the desert surface with the hottest layer next the surface, a highly unstable condition and hence usually seen only with conditions of quiet air. On our modern surfaced highways this phenomenon is nearly always to be seen on cold but sunny and quiet days where one comes up a grade to a flat section of the highway, the effect being the mirrored sky as seen in a pool of water. This phenomenon, also fairly characteristic of the polar areas, is one of relatively short distances that are usually to be measured in rods, whereas the looming of the high latitudes is one not of rods or even miles, but of tens or hundreds of miles. The deviation of the light rays from the object to the eye in the case of inferior mirage is always in a curve that is convex downward toward the earth's surface, whereas with superior mirage it is by contrast concave downward. With the rays bent convexly toward the earth as in desert mirage, the height of the eye sets the very low limit to the distance of objects seen as a consequence of it, and the image is an inverted virtual one, usually of the sky, though sometimes of rocks and trees. Superior mirage is due to the presence of a stratum of warmer layers of air *above* colder ones at some considerable distance above the surface, what is known to meteorologists as an *inversion:* i.e., inversion of temperature gradient so that the layers within the stratum are warmer

upward. The distance of the objects seen will be determined by the altitude of the inversion layers and by the extreme differences in temperature which they represent (Fig. 1).

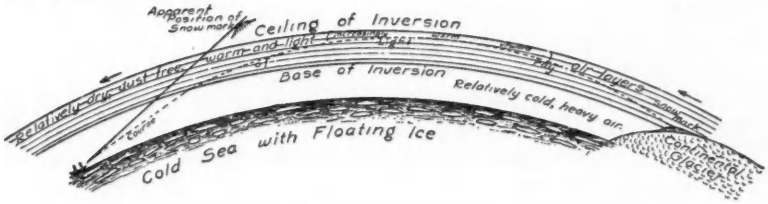
Apparent position of Snow mark
Ceiling of Inversion
Base of Inversion
Relatively cold, heavy air
Cold Sea with Floating Ice

FIG. 1—Superior mirage or "looming" (inversion stratum increasingly warm upward).

Studies made with *ballons-sondes* which carry self-recording instruments have shown that the inversions which are most characteristic of the polar regions, and particularly within the areas about the great continental glaciers, generally begin either at a height of from 1000 to 1500 meters or else in the neighborhood of 200 meters or more. Thus Alfred Wegener found for the Danmarks-Havn station in Northeast Greenland centred around a rather general altitude of 1200 meters a stratum of relatively warm and dry air layers, with another below 500 meters and sharpest below 200 meters.[1] De Quervain[2] from use of pilot balloons found the same outward-streaming air currents at about 1000 meters elevation. In East Greenland Kopp confirmed an inversion at different seasons of the year and representing a difference of temperature of as much as 25° C. culminating near the elevation of 1000 meters.[3] Hergesell[4] with use of *ballons-sondes* on the *Princess Alice* over the Greenland Sea between Iceland and Greenland found on certain days no inversions, but on others and rather more frequently he found strong inversions generally culminating at or below 1000 meters, though sometimes up to 1500 or even 2100 meters altitude.

In the Antarctic from balloon studies over the Weddell Sea Barkow found inversions strong in autumn and winter below 500 meters, in winter up to 6° C., in autumn there was a strong one above 2000 meters amounting to over 6° C. and in winter smaller ones at 1000–1500 meters and at

[1] A. Wegener, "Drachen- und Fesselballonaufstiege," *Med. om Grönl.*, Vol. 42, 1909, p. 61.

[2] A. De Quervain, *Beiträge z. Physik d. freien Atmosphaere*, Vol. 5, 1913, pp. 132–158.

[3] Else Wegener, *Alfred Wegeners Letzte Grönlandfahrt*, Leipzig, 1932, p. 276.

[4] H. Hergesell, "Aerologische Studien im Arktischen Sommer," *Beitr. z. Physik d. freien Atmosphaere*, Vol. 6, 1913, pp. 224–261.

1800 meters.[5] An average of 50 ascents of captive balloons made throughout the winter gave an average inversion of 7.1° C. Simpson with ascents of *ballons-sondes* at Cape Evans found for the winter months an inversion extending up to 1000 meters and representing an increase of 10° C., but of this there was no evidence during the summer.[6]

It has already been pointed out that because of the high altitude of the inversion layer the object seen must be at a very considerable distance, if the observer is near sea-level, and the object seen is beyond the normal horizon. Moreover, since the range of temperature difference within the inversion layer is limited to a few degrees centigrade in a vertical distance of 500 meters or more, the refraction curve of the ray must be one of very long radius of curvature—the curve must be very flat. It is a further peculiarity of the real image that enters the eye that it is expanded vertically so that objects are considerably distorted when seen by superior mirage. It follows from both these conditions that the object can be seen only from within a very limited area—the segment of a ring. Obviously at points nearer the object the curvature of the rays would not be sufficient to bring them down to the earth. This peculiarity of looming is little appreciated even by those who have given some attention to the subject, more especially by Mawson.[6a]

It is characteristic of the high superior mirage, that above 1000 meters, that there is usually a band which separates the raised image from the true horizon (see Fig. 5, p. 237). In the case of the lower type, that near 200 meters, it may be the top of the object which is truncated in the image (Fig. 2).

The not uncommon phenomenon of superior mirage has been responsible for many false estimates of remoteness of newly discovered land features which have been seen by explorers within the polar regions, combined as it has been with the underestimates due to the unusual clarity of the atmosphere. In many cases of snow-covered lands there is not enough of individual character in the coastal features to permit of identification from different ship positions, and in such cases the newly discovered lands have of necessity been placed upon the maps on the basis of their direction and estimated distance, and as a consequence they are often as much as forty

[5] E. Barkow, "Die Ergebnisse der meteorologischen Beobachtungen der Deutschen Antarktischen Expedition 1911–1912, Herausgeg, v. K. Knoch," *Veröff. d. preusz. Meteor. Instituts* (v. Ficker), bd. vii, no. 6, Berlin, 1924, pp. 81–82.

[6] G. C. Simpson, *Brit. Ant. Exped., Meteorology, 1910–13,* Calcutta, 1919, Vol. 1, "Discussion," p. 276.

[6a] "Wilkes's Antarctic Landfall," *Proc. Royal Geog. Soc.,* South Australian Br. Sess., 1932–33, pp. 69–113.

or fifty geographical miles too near when this is due to the atmospheric clarity alone, but as much as two hundred up to three hundred geographical miles when due to high superior mirage. It has been in this way that many of the puzzling confusions have arisen, and explorers have repeatedly and quite unjustly been brought into discredit by later ship captains, who have arrived off these shores when sea-ice conditions were more favorable and have permitted of nearer approach, or when atmospheric conditions were less deceptive. To errors in latitude were in the case of earlier explorers added the considerable uncertainty in longitude determination, which before the advent of radio time-signals were often in error by two or more degrees, even after checking the chronometer.

A considerable number of such puzzles of discovery have now been cleared up, and the early explorers have been cleared of suspicion or discredit. These have been discussed by the author in several papers.[7] Perhaps the most interesting of these are the "Croker Mountains" which in Lancaster Sound appeared at a distance of about thirty miles to bar the way into a Northwest Passage to Captain, later Sir John, Ross in 1818, in all probability the snow-covered heights of North Somerset Island fully two hundred geographical miles distant; the supposed "Parry Mountains" of Captain Sir James Clark Ross upon the Ross Barrier to the eastward of

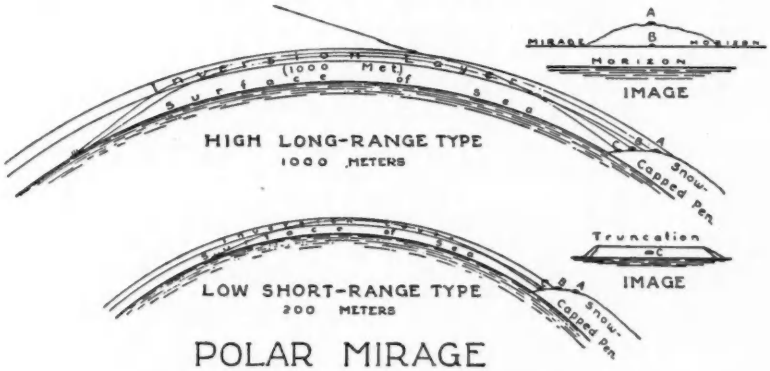

FIG. 2—Different varieties of superior mirage.

[7] "Wilkes Land Rediscovered," *Geogr. Rev.*, Vol. 22, Oct., 1932, pp. 632–655. "The Eastern Landfalls of Wilkes within the Australian Sector of the Antarctic," *Geogr. Journ.*, Vol. 81, June, 1933, pp. 538–540. "Visibility and the Discovery of Polar Lands," *Geografiska Annaler*, 1933, Hefts 2 and 3, pp. 217–224, figs. 6. "Discovery of a New Sketch of Cape Hudson in the Antarctic," *Geogr. Rev.*, Vol. 24, Jan., 1934, pp. 115–117, figs. 2.

Ross Island in the Antarctic at an estimated distance of perhaps twenty-five miles, and with little doubt the high range which includes Mount Hamilton and Mount Albert Markham at a distance of nearly three hundred geographical miles (Fig. 3); the Wilkes Land landfalls of Lieutenant, later Rear Admiral, Charles Wilkes in his discovery of the Antarctic Continent in 1840.[8]

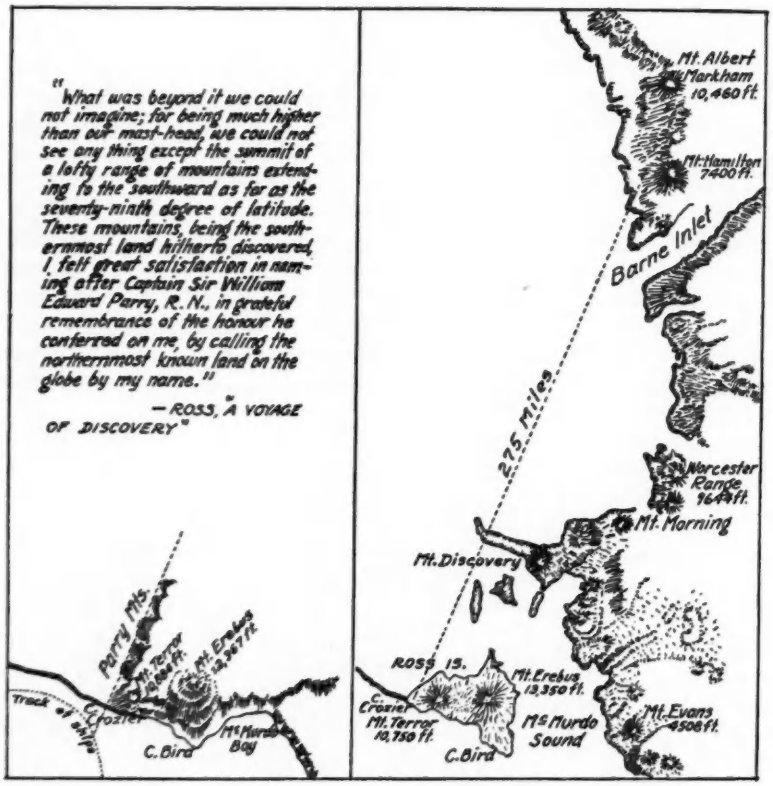
"What was beyond it we could not imagine; for being much higher than our mast-head, we could not see any thing except the summit of a lofty range of mountains extending to the southward as far as the seventy-ninth degree of latitude. These mountains, being the southernmost land hitherto discovered, I felt great satisfaction in naming after Captain Sir William Edward Parry, R.N., in grateful remembrance of the honour he conferred on me, by calling the northernmost known land on the globe by my name."
— ROSS, "A VOYAGE OF DISCOVERY"
Parry Mts
Mt. Terror 10,868 ft.
Mt. Erebus 12,367 ft.
Track of ships
C. Crozier
Mt. Murdo
C. Bird
Bay
Mt. Albert
Mt. Markham
10,460 ft.
Mt. Hamilton
7400 ft.
Barne Inlet
275 miles
Worcester Range
9644 ft.
Mt. Morning
Mt. Discovery
ROSS I.
Mt. Erebus
13,350 ft.
C. Crozier
Mt. Terror
10,750 ft.
Mc Murdo Sound
C. Bird
Mt. Evans
4508 ft.
ROSS, 1841
SCOTT, 1910–13.
SCALE THE SAME FOR BOTH MAPS

FIG. 3—The "Parry Mountains" of James Clark Ross in supposed and real positions.

[8] The first clear discovery of festland on the Antarctic Continent, now that Ellsworth has shown Graham Land to be a peninsula and not an archipelago as sup-

As regards the existence of Wilkes Land, Ross, Borchgrevink, Scott, Shackleton and Mawson, all British Antarctic explorers, strove to discredit Wilkes and each succeeded in showing that some one or more of the landfalls were not in the locations where placed upon the map by Wilkes. With the exception of Wilkes's "Termination Land," the westernmost of all his landfalls and apparently an ice rather than a land feature, the Wilkes landfalls have now been shown to exist, but to be in more remote positions, that is to say, they lay farther to the south than he supposed, due to the underestimate of distance by in most cases forty or fifty miles, but in the easternmost landfalls by as much as two hundred geographical miles as a consequence of high superior mirage (Fig. 4). This is notably true of Wilkes's Cape Hudson, now clearly shown to be the high glacier-covered land back of Cape Freshfield in Mawson's King George Land through the happy comparison of the sketches made by Wilkes in 1840 and by the British Captain J. R. Stenhouse in the *Aurora* three quarters of a century later. These were made from near the same observation point and seen in the same direction[9] (Fig. 5). The direction was also happily revealed in spite of the deranged compasses by the simultaneous sighting of the known Young Island (230 geographical miles away) directly east as fixed by latitudes.[10] The *Aurora* had for months been drifting in positions much nearer to the Cape Freshfield area without once sighting it.

Of all the attacks made upon Wilkes those by Mawson have been the most persistent.[11] Mawson has made much of the fact that he sailed over Cape Carr, a landfall of Wilkes in longitude 133° E. (Fig. 6). He could not of course have foreseen that he was nearly a score of years later and for almost identical reasons to sail over nearly three hundred miles of his own mapped land of a year earlier (Fig. 7). There is almost a complete parallel

posed by Wilkins, was Palmer Land mapped by the American, Captain Nathaniel Brown Palmer, more than a decade before Biscoe sighted Enderby Land. But it was Wilkes who by a series of landfalls distributed throughout a distance of 1500 miles and by soundings along the front to discover a continental shelf, first proved its continental character.

[9] W. H. Hobbs, "Discovery of a New Sketch of Cape Hudson in the Antarctic," *Geogr. Rev.*, Vol. 24, Jan., 1934, pp. 115–117, Figs. 1–2.

[10] This point has been missed by Wordie (J. M. Wordie, The Ross Sea Drift of the "Aurora" in 1915–1916, *Geogr. Journ.*, Vol. 58, 1921, pp. 219–224.

[11] Sir Douglas Mawson, D.Sc., B.E., *The Home of the Blizzard*, etc., 1914. Vol. I, pp. 646–647. Douglas Mawson, Kt., D.Sc., F.R.S., *The Unveiling of Antarctica* (Presidential Address), Australian and New Zealand Association for the Advancement of Science, Rept. of the Melbourne Meeting, 1935, pp. 15–19. D. Mawson, Kt., etc., "Wilkes's Antarctic Landfalls," *Proc. Roy. Geogr. Soc.*, South Australasian Branch, Sess. 1932–33, pp. 69–113, 3 maps.

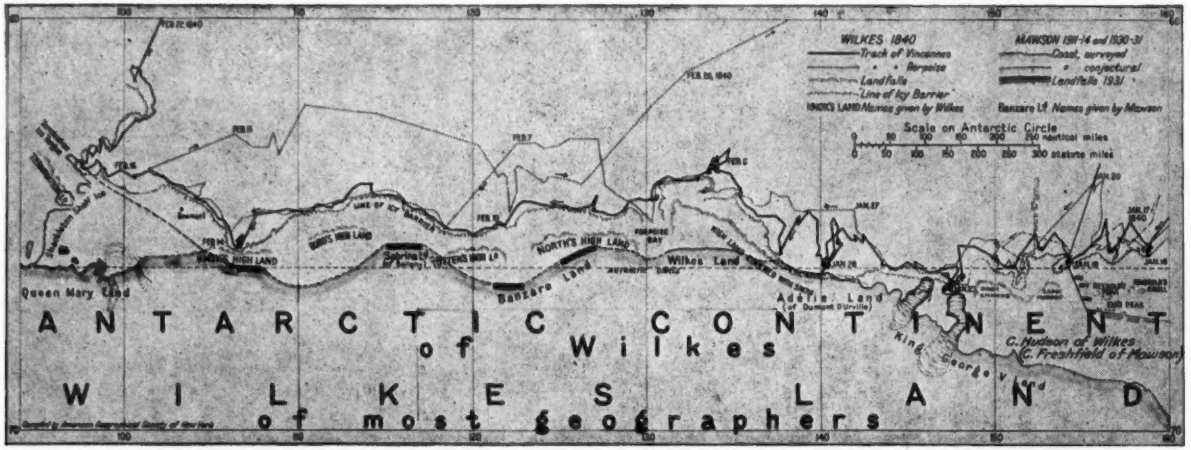

FIG. 4—Comparison of landfalls of Wilkes in 1840 with those of Mawson in 1911–14. Courtesy of the *Geographical Review* published by the American Geographical Society of New York.

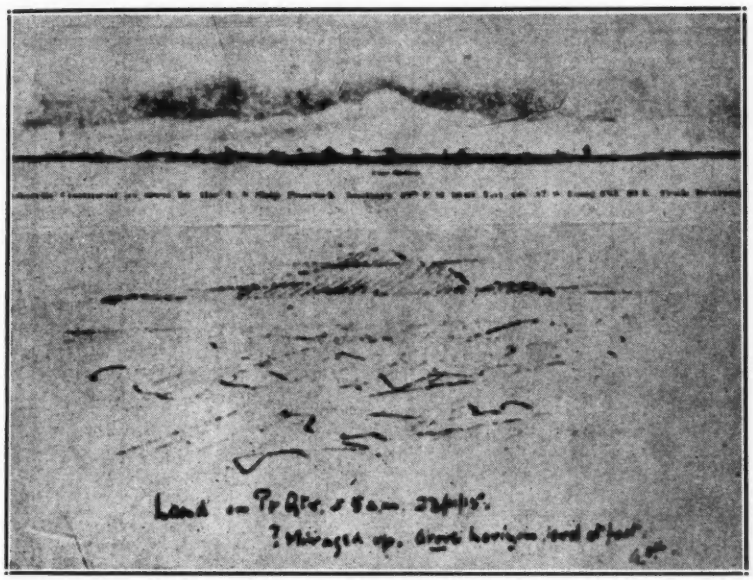

FIG. 5—Sketches of Cape Hudson (Cape Freshfield of Mawson) by Wilkes from the *Peacock* in 1840 and by Ninnis on the *Aurora* in 1915.

between the coast of Wilkes Land as mapped by Wilkes as Antarctic Continent in 1840 and Mawson's wrongly renamed Banzara Land in 1914.

One additional example needs to be cited even though its problem has not yet been cleared up. In 1906 Peary and his Eskimo companions all saw clearly an extended mountainous snow-covered land to the northwest of Cape Colgate in southwestern Grant Land; and a few days later this land was again clearly seen by all of them from above Cape Thomas Hubbard in Axel Heiberg Land. In 1914 MacMillan and Fitzhugh Green with Eskimo companions all saw this land in the same direction from above Cape Thomas Hubbard extending through a wide arc of the horizon. Peary had named this land Crocker Land and had estimated its distance to be one hundred and thirty miles. MacMillan and Green in 1914 sledged that distance in the direction indicated without finding it, though they all saw it again on one occasion on the journey. If Peary made the usual underestimate of explorers in such instances, the land probably lies within the vast unexplored area between the track of the *Norge* and the route of Wilkins in his plane in the flight of 1928 (Fig. 8).

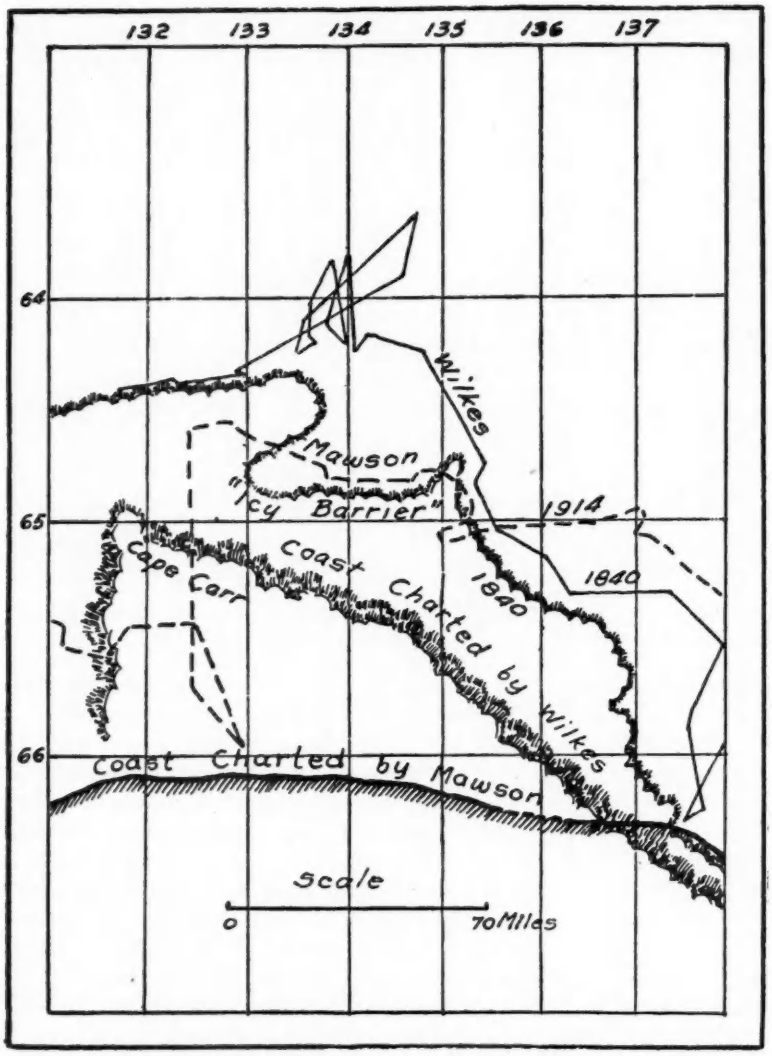

FIG. 6—Mawson's map to show how he sailed over Wilkes's Cape Carr.

The above are a few, but yet significant examples of the confusions which have arisen in the past in the discovery of polar lands due to peculiarities

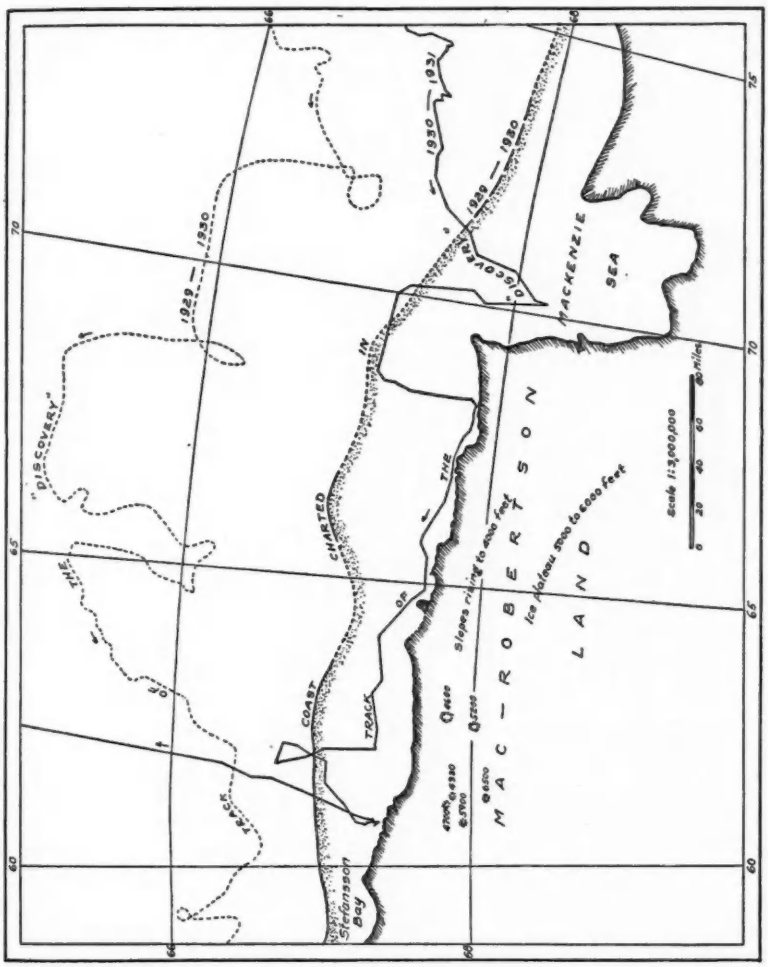

FIG. 7—Charted shore lines of Mawson's MacRobertson Land as mapped by him in 1929–30 and in 1930–31, when with better ice conditions he was able to get nearer to the coast. His two maps have here been combined.

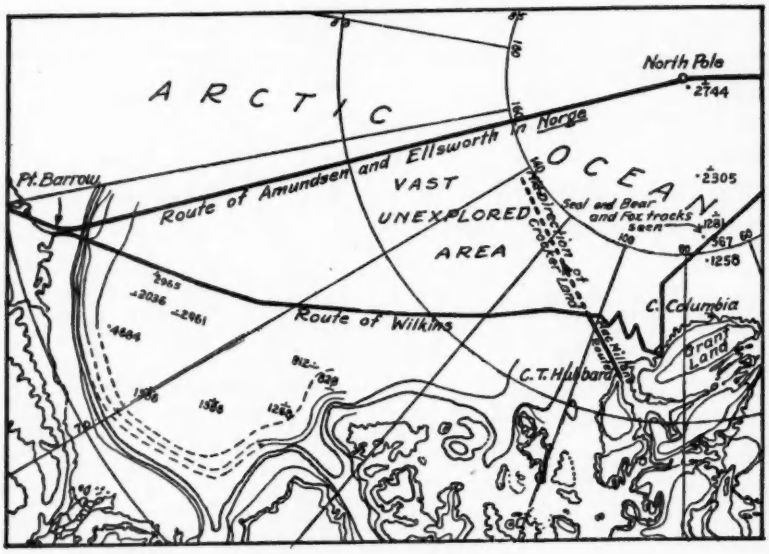

FIG. 8—Map to show the probable position of "Crocker Land."

of visibility within these polar regions. They do not, of course, by any means exhaust the many varieties of mirage which are peculiar to those regions, but only those which have more serious import as concerns the discovery of polar lands.

University of Michigan,
May, 1937.